

Revising the Borgatti-Everett core-periphery model: Inter-categorical density blocks and partially connected cores

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ABSTRACT

Borgatti and Everett's model (2000) remains the prevailing standard for identifying categorical core-periphery structures in empirical networks, yet this method poses two significant issues. The first concerns the handling of inter-categorical ties—those linking core and periphery actors. The second problem is the model's definition of the ideal core as a complete block or clique, which can be overly stringent in practical applications. Building on advancements in direct blockmodeling, we propose modifications to address these shortcomings. To better handle inter-categorical ties, we replace the traditional cell-wise correlation approach with one based on exact- and minimum-density blocks. To relax the constraint of a fully connected core, we introduce the p-core, a proportional adaptation of the k-core/k-plex cohesive subgroups, providing greater flexibility in defining the level of cohesion required for core membership. We illustrate the advantages of these enhancements using both classic network examples and synthetic networks.

Introduction

Few concepts in network analysis are as universally applicable as the core-periphery structure. Its origins can be traced back to Prebisch's (1950) seminal work and subsequent developments in post-war heterodox political economy (see Love, 1980; Nordlund, 2018). With notable pre-cursors in Galtung's work on "feudal interaction structures" (Galtung, 1971, see also 1966a, 1966b), the concept was further refined

by early blockmodeling scholars (Breiger et al., 1975; Mullins et al., 1977; White et al., 1976). Today, core-periphery structures are applied across diverse domains, from international relations to protein interactions.³

In contemporary network analysis, a core-periphery structure represents a distinct meso-scale configuration where network members are partitioned into two disjoint subsets: the core and the periphery, each defined by unique relational characteristics. The core comprises tightly

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³ Within political science, the core-periphery concept finds application in a diverse array of studies, spanning from analyses of political participation (Galtung, 1964; Langholm, 1971) to examinations of international relations (Berman, 1974; Chan, 1982; Dominguez, 1971; Galtung, 1966a; Gochman and Ray, 1979; Snyder and Kick, 1979; Thompson, 1981). In human geography, core-periphery dynamics are observed in regional studies (Friedmann, 1966; Hanink, 2000; Kaupila, 2011), transportation geography (Gleditsch, 1967; Goetz and Sutton, 1997), communication studies (Sun and Barnett, 1994), and various urban studies contexts, including intra-city (Uzzi and Spiro, 2005), inter-city (Wellhofer, 1989), and world city levels (Alderson and Beckfield, 2004). Sociology explores core-periphery structures within organizations and workgroups (Crowston et al., 2006; Cummings and Cross, 2003), exchange dynamics among Papua New Guinea hunters (Healey, 1990), community prevention programs (Feinberg et al., 2005), stakeholder networks (Boutilier, 2011), ecosystem-related advice networks in locations such as Chile (Giuliani and Bell, 2005), Ghana (Isaac et al., 2007), and Stockholm (Ernstson et al., 2008), and even in the regulation of accounting and auditing standards in Canada (Richardson, 2009). Sociology's widespread adoption of the core-periphery concept may reflect the migration patterns of sociology PhD students, which resemble a core-periphery structure (Burris, 2004). In biology, the human metabolic network has been conceptualized as a core-periphery structure (Zhao et al., 2007), as have citation patterns among biology scholars (Mullins et al., 1977) and the co-citation behavior of academic scholars in general (White, 1990; Zuccala, 2006). If the very concept of core-periphery structures spreads akin to an infectious disease, its diffusion may indeed resemble a core-periphery structure (cf. Christley, 2005).

interconnected actors (or nodes), while the periphery consists of actors lacking direct links among themselves. Additionally, a core-periphery structure presupposes overall network connectivity, with sufficient inter-categorical ties linking core and peripheral actors, thereby integrating the periphery into the network through its connections to the core (Borgatti and Everett, 2000, p. 382).

Despite the development of various alternative models in recent years (e.g. Boyd et al., 2006, 2010; Cucuringu et al., 2016; Della Rossa et al., 2013; Gallagher et al., 2021; Kojaku and Masuda, 2017, 2018; Lip, 2011; Rombach et al., 2014; Tang et al., 2019; van Lidth de Jeude et al., 2019), Borgatti and Everett's (2000) model remains widely used for identifying categorical core-periphery structures in empirical data (e.g., Rombach et al., 2017, p. 620). This model formalizes the core as a fully connected (complete) block and the periphery as a fully disconnected (null) block, while optionally allowing users to specify desired densities for the two blocks connecting the core and periphery. It then uses a partitioning search algorithm alongside a goodness-of-fit indicator to seek the optimal core-periphery partition.

This model, however, is troubled by two significant issues. The first concerns the treatment of inter-categorical ties—connections between core and periphery actors—when determining the optimal partition. Borgatti and Everett (2000, p. 382ff) offer two options: either disregard inter-categorical ties and focus solely on intra-categorical ties, or specify desired densities for inter-categorical blocks. While the first approach is often seen as the most practical (Boyd et al., 2006, p. 167), it completely omits crucial information about the relationships between core and periphery actors—an essential element of any core-periphery structure (see, e.g., Della Rossa et al., 2013; Rombach et al., 2017). The second option, which uses the density of inter-categorical blocks as a proxy for overall connectivity, can be impractical and is poorly implemented. Since the model correlates binary matrices (0's and 1's) with ideal-pattern matrices featuring non-integer values, the goodness-of-fit measure becomes unreliable, yielding potentially misleading results.

The second issue involves defining the ideal core as a fully connected block, where all core members are directly connected with each other. While complete connectedness aligns with the concept of a densely interconnected core, this requirement can be excessively stringent in practice, especially as networks—and their potential cores—grow in size. Even small networks considered ideal-type core-periphery structures, like the Galtung network (see below), fail to meet this exacting requirement. Although this issue has been acknowledged by the authors (Everett and Borgatti, 2000), alternative representations of the ideal core pattern, allowing for some leniency, have yet to be implemented.

To address these shortcomings, this paper builds on recent advancements in direct blockmodeling (Nordlund, 2020). We propose modifications to the Borgatti-Everett model to better handle inter-categorical ties and relax the constraint of a fully connected core. For inter-categorical ties, we introduce exact- and minimum-density ideal blocks. The exact-density blocks approach stays true to the original model by penalizing partitions that deviate from the specified inter-categorical density, while the minimum-density blocks approach is more pragmatic, penalizing only those partitions that fall short of the minimum required density. To address the issue of a fully connected core, we implement the *p*-core, a proportional adaptation of the *k*-core/*k*-plex cohesive subgroups, offering greater flexibility in defining the cohesion needed for core membership.

Having implemented both the classical Borgatti-Everett core-periphery model and the modifications we propose in R and Socnet.se, the latter constituting a novel console application designed for direct blockmodeling that supports all core-periphery models discussed in this article, we demonstrate the benefits of our proposed modifications using both classical and synthetic network examples. The article concludes with a brief discussion of the limitations of our approach and suggestions for future research directions.

The Borgatti-Everett core-periphery model

The Borgatti-Everett model takes as input an observed (potentially valued) monopartite network and outputs a binary vector $\mathbf{v} = \{v_1, v_2, \dots, v_n\}$ of length n (the number of actors in the network), together with a goodness-of-fit measure. Each entry v_i is either 1 or 0, where 1 indicates that the actor belongs to the core, and 0 indicates that the actor belongs to the periphery.⁴ To determine the network partition that best fits a core-periphery structure, the model compares the network to an ideal-pattern representation, which consists of a fully connected core block and a periphery block with no internal ties. This comparison is performed using a suitable combinatorial optimization technique⁵ and a goodness-of-fit measure, which evaluates how closely the partition resembles the ideal core-periphery structure.

Let $\mathbf{A}$ be the $n \times n$ adjacency matrix where $a_{ij} = 1$ if actor i sends a tie to actor j , and $a_{ij} = 0$ otherwise (self-loops, a_{ii} , are typically undefined). For a given partition $\mathbf{v}$, the adjacency matrix $\mathbf{A}$ is compared with a block model or ideal-pattern matrix $\mathbf{B}(\mathbf{v})$, which is constructed as follows:

$$b(\mathbf{v})_{ij} = \begin{cases} 1 & \text{if } v_i = v_j = 1, \quad i \neq j \\ d \in [0, 1] \text{ or NA} & \text{if } v_i = 1 \text{ XOR } v_j = 1 \\ 0 & \text{otherwise,} \end{cases}$$

where "XOR" denotes the "exclusive or" operator. The parameter d can be set to the expected density of the inter-categorical blocks (i.e., the blocks connecting core and periphery), but it is often treated as missing data (NA, for not applicable), following the authors' recommendation.

To illustrate the model, Fig. 1A and B show a network partition and its corresponding ideal-pattern block model. For ease of visualization, the adjacency matrix $\mathbf{A}$ is often rearranged into four blocks:

$$\mathbf{A} = \begin{pmatrix} \mathbf{A}^{cc} & \mathbf{A}^{cp} \\ \mathbf{A}^{pc} & \mathbf{A}^{pp} \end{pmatrix},$$

where $\mathbf{A}^{cc}$ (top left) and $\mathbf{A}^{pp}$ (bottom right) represent the core and periphery blocks, respectively. $\mathbf{A}^{cp}$ (top right) contains the core-to-periphery ties, and $\mathbf{A}^{pc}$ (bottom left) represents the periphery-to-core ties.

The Pearson correlation coefficient r between the cell values in $\mathbf{A}$ and $\mathbf{B}(\mathbf{v})$ serves as both the objective function and goodness-of-fit statistic:

$$r = \frac{\sum (a_{ij} - \bar{a})(b_{ij}(\mathbf{v}) - \bar{b}(\mathbf{v}))}{\sqrt{\sum (a_{ij} - \bar{a})^2 \sum (b_{ij}(\mathbf{v}) - \bar{b}(\mathbf{v}))^2}}$$

where $\bar{a}$ and $\bar{b}(\mathbf{v})$ represent the mean values of the matrices $\mathbf{A}$ and $\mathbf{B}(\mathbf{v})$, respectively. The partition(s) that yield the highest r value are considered the model's optimal output. Theoretically, the maximum value of r is 1, which would indicate a fully connected core, an empty periphery, and, if specified, inter-categorical blocks matching the pre-specified density d .

To observe the model in practice, we applied it to four small undirected networks: *BEfig1* (a toy example used by Borgatti and Everett, 2000), *Baker* (a network based on journal citation data by Baker (1992), here dichotomized and max-symmetrized by Borgatti and Everett (also Nordlund, 2018, p. 360ff, Nordlund, 2016, p. 167ff)), *Galtung* (a feudal interaction structure from Galtung, 1971), and the *Zachary* Karate Club network (Zachary, 1977), which features two emerging groups resulting

⁴ In addition to this discrete core-periphery model, Borgatti and Everett also introduce a continuous model where "coreness" is a nodal continuous measure (Borgatti and Everett, 2000, p. 385ff). This article, however, focuses specifically on the discrete model.

⁵ An exhaustive search that explores all possible network partitions is feasible only for small networks. For larger networks, more efficient (yet not exhaustive) search algorithms are employed instead. The Socnet.se software client provides two such search algorithms.

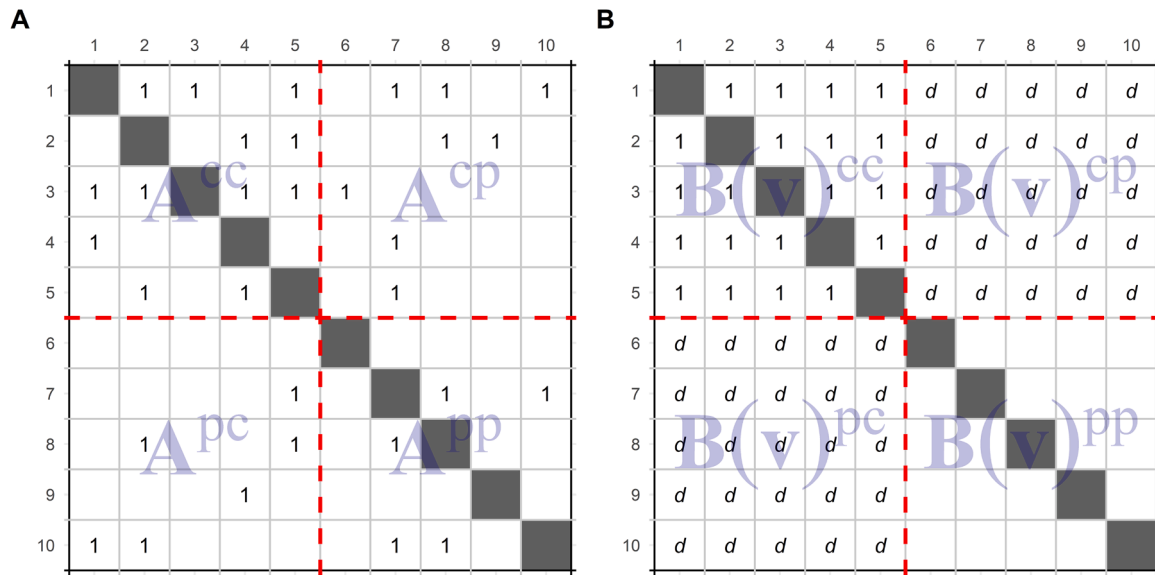


Fig. 1. Example of a network partition and its ideal-pattern matrix. (A) Adjacency matrix representing a core-periphery partition. A two-group partition produces four distinct blocks: core-to-core ties (A^{cc} , top left), core-to-periphery ties (A^{cp} , top right), periphery-to-core ties (A^{pc} , bottom left), and periphery-to-periphery ties (A^{pp} , bottom right). (B) The ideal-pattern matrix corresponding to this partition, illustrating a fully interconnected core, an empty periphery, and inter-categorical blocks based on a pre-defined density (d). Blank cells indicate zeroes, representing the absence of ties.

from a split within the club. Table 1 presents descriptive statistics for these networks, and Fig. 2 visually displays the partitions retrieved with information from the inter-categorical blocks treated as missing data.

For *BEfig1*, the model identifies actors 1–4 as the core, with an r value of 1, indicating a perfect match with the ideal core-periphery structure. In the *Baker* network, 7 of the 20 actors are assigned to the core, with one core-to-core tie missing and four periphery-to-periphery ties, resulting in an r value of 0.8596. The optimal partition for *Galtung* assigns actors C1, C2, C3, and C4 to the core, but the lack of two core-to-core ties leads to an r value of 0.7947.⁶ Finally, in *Zachary*, 9 of the 34 actors are classified as the core, with an r value of 0.5432, reflecting a significant deviation from the ideal-pattern matrix

The issue with the treatment of inter-categorical ties

While the Borgatti-Everett model remains widely used, its treatment of inter-categorical ties raises concerns. In its most common implementation, the model overlooks these ties by treating the parameter d as missing ($d = NA$), thus only focusing on intra-categorical ties (an issue

we discuss further later on). When the density parameter d is set to a particular value ($d \in [0, 1]$), the model examines not only dense cores and sparse peripheries but also the presence or absence of ties between core-periphery pairs.

Despite the theoretical benefits of this last approach, the Borgatti-Everett model exhibits limitations when d is set to values other than the extremes of complete presence ($d = 1$) or absence ($d = 0$) of inter-categorical ties. For instance, in the *BEfig1* example, setting $d = 1/3$ yields a lower r -value (0.6686) compared to when d is treated as missing, even though the partition meets all model criteria—a fully connected core, an empty periphery, and inter-categorical blocks with the specified density.

This issue persists across all the networks we tested: even when specifying that the ideal inter-categorical density should match the empirically observed density (when inter-categorical ties are ignored), the resulting goodness-of-fit consistently decreases, despite a perfectly specified inter-categorical density. The root of this paradox lies in how the Borgatti-Everett model calculates the density of inter-categorical blocks. By correlating each inter-categorical cell (binary values of 1's and 0's) with the target density, the model should, in theory, find the optimal partition at the specified density. However, the goodness-of-fit score is significantly diminished as a result (see Panel A in Fig. 3).

Table 1
Descriptive statistics of the four networks.

Network	Actors	Ties	Density	Average degree	Transitivity	Diameter
BEfig1	10	14	0.311	2.800	0.419	3
Baker	20	59	0.311	5.900	0.498	3
Galtung	13	13	0.167	2.000	0.000	4
Zachary	34	78	0.139	4.588	0.256	5

⁶ While the optimal *Baker* partition exhibits a total of five deviations from its ideal core-periphery model, compared to the two deviations in the optimal *Galtung* partition, the relative 'cost' for lacking core ties in the Galtung solution is higher than the corresponding 'cost' of the additional intra-peripheral ties and the missing core tie in the *Baker* partition. This discrepancy arises from the absolute size differences of the respective networks, the relative sizes of their core and periphery partitions, and the inherent bias of point-biserial correlations (Kemery et al., 1988; Nordlund, 2020, p. 131).

Exact-density ideal blocks

To address the issue with the current implementation of inter-categorical densities, the Borgatti-Everett model can be improved by replacing the cell-wise correlation-based approach with one based on *exact-density ideal blocks*. This approach ensures that the goodness-of-fit statistic r more accurately reflects the alignment with pre-specified criteria. By avoiding the use of non-integer values in the ideal-pattern matrix, the approach eliminates the distortion that arises when binary networks are compared with ideal-pattern matrices containing non-integer values.

The exact-density blocks approach works by matching the observed inter-categorical blocks with an ideal-pattern representation containing a specified number of 1's, determined by the desired density d , with the remaining cells filled with 0's. In our implementation, the two off-diagonal blocks in A (A^{cp} and A^{pc}) are first vectorized and then sorted

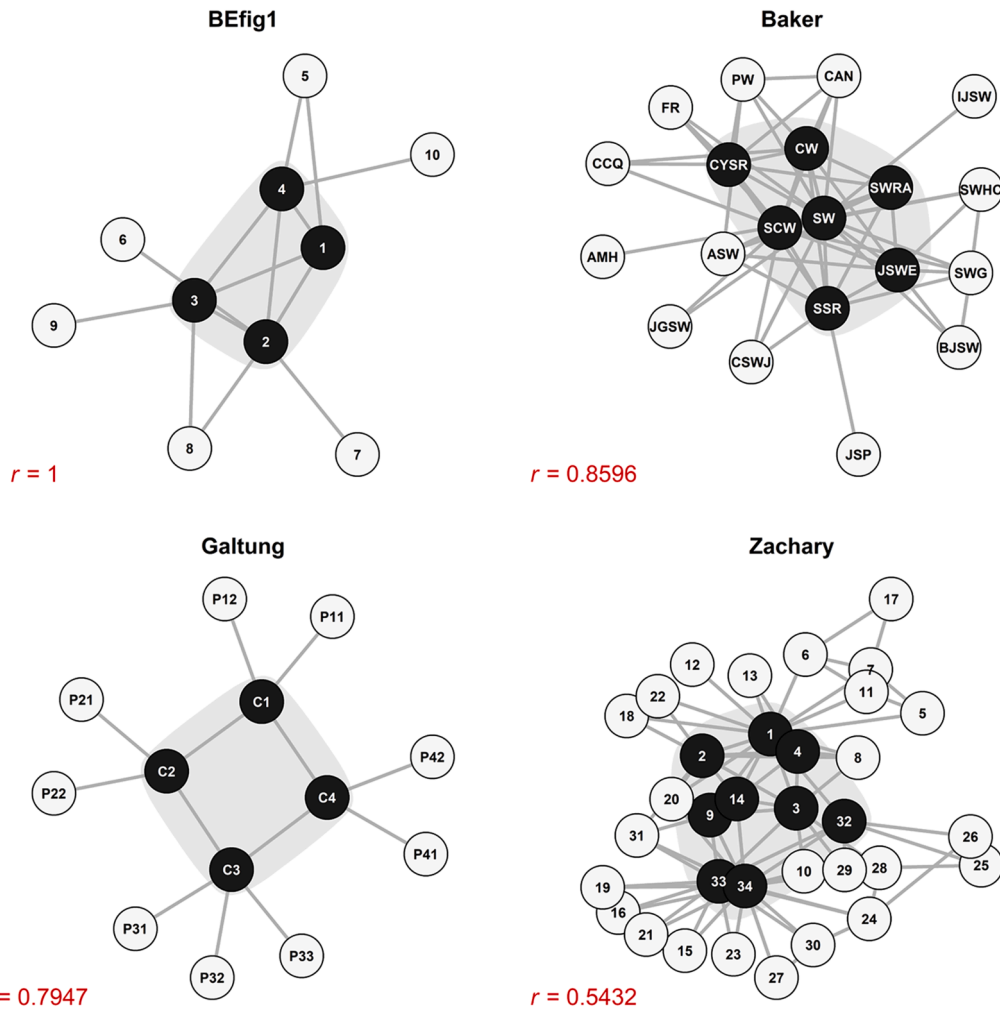


Fig. 2. Solutions using the Borgatti-Everett core-periphery method. Inter-categorical ties ignored in these solutions ($d = \text{NA}$).

in descending order: $\mathbf{a}^{\text{cp}1}$ and $\mathbf{a}^{\text{pc}1}$. These sorted vectors are then matched with an ideal-pattern vectors $\mathbf{b}(\mathbf{v})^{\text{cp}}$ and $\mathbf{b}(\mathbf{v})^{\text{pc}}$ where:

$$b(\mathbf{v})_i^{\text{cp}} = \begin{cases} 1 & \text{if } i \leq \lfloor d \cdot |\mathbf{A}^{\text{cp}}| \rfloor, \\ 0 & \text{otherwise,} \end{cases}$$

$$b(\mathbf{v})_i^{\text{pc}} = \begin{cases} 1 & \text{if } i \leq \lfloor d \cdot |\mathbf{A}^{\text{pc}}| \rfloor, \\ 0 & \text{otherwise,} \end{cases}$$

where $|\mathbf{A}^{\text{cp}}|$ and $|\mathbf{A}^{\text{pc}}|$ represent the number of cells in the inter-categorical blocks, and $\lfloor x \rfloor$ indicates rounding to the nearest integer. Note that since $|\mathbf{A}^{\text{cp}}| = |\mathbf{A}^{\text{pc}}|$, $\mathbf{b}(\mathbf{v})^{\text{cp}} = \mathbf{b}(\mathbf{v})^{\text{pc}}$ unless different densities are specified for the core-periphery and periphery-core blocks. Here, we assume the researcher seeks the same density for both blocks ($d^{\text{cp}} = d^{\text{pc}} = d$). However, software like UCInet or Socnet.se allow for these densities to be specified separately. Also note that this modification does not alter the treatment of values in the core and periphery blocks ($\mathbf{B}(\mathbf{v})^{\text{cc}}$ and $\mathbf{B}(\mathbf{v})^{\text{pp}}$).

As shown in Fig. 3, Panel B, the exact-density blocks approach ensures that when $d = 1/3$, the *BEfig1* example produces an r value of 1 for the optimal partition, indicating perfect alignment with the ideal core-periphery structure—a fully connected core, an empty periphery, and inter-categorical blocks matching the specified density.

What if the partition closely matches, but does not perfectly fulfill, all criteria? This scenario can be illustrated using *BEfig1* with d values close to $1/3$. For instance, choosing $d = 0.2833$ ($1/3 - 0.05$) generates an ideal-pattern representation with $\lfloor 0.2833 \cdot 24 \rfloor = 7$ 1's followed by 17

0's. In this case, each inter-categorical block contributes 7 pairs of (1,1), 16 pairs of (0,0), and one (1,0) pair, resulting in an r value of 0.9484 (see Panel D). With $d = 0.3833$ ($1/3 + 0.05$), the representation consists of 9 ones followed by 15 zeroes, yielding an r value of 0.9485 (see Panel F). Both values, 0.9484 and 0.9485, indicate a high degree of alignment between the observed and ideal values, albeit with slight deviations from perfect criteria fulfillment.

Panels A, C, and E present the results using the standard Borgatti-Everett implementation. As can be observed, inter-categorical blocks introduce distortion into the point-biserial correlation, resulting in r -values that poorly capture the level of agreement between the observed network and its ideal-pattern representation.

The improved accuracy of r in reflecting the degree of alignment with the pre-specified requirements is further demonstrated in other networks. Fig. 4 compares the r values obtained for solutions with d ranging from 0 to 1 in increments of 0.05 using different procedures. The top panels show results from the standard approach, while the middle panels display those using exact-density ideal blocks.

One key observation from Fig. 4 is that with the exact-density blocks method, the r -value reaches its maximum when the specified d -value matches (or closely approaches) the block density found in the partition retrieved when inter-categorical ties are ignored (indicated by the dashed red vertical line). This pattern is not observed with the original approach. For example, in the *Galtung* network, the partition obtained when ignoring inter-categorical ties contains 9 inter-categorical ties out of 27 possible (25 %). When d is set to 0.25, the exact-density method produces the maximum r -value (0.9165) for this network. In contrast,

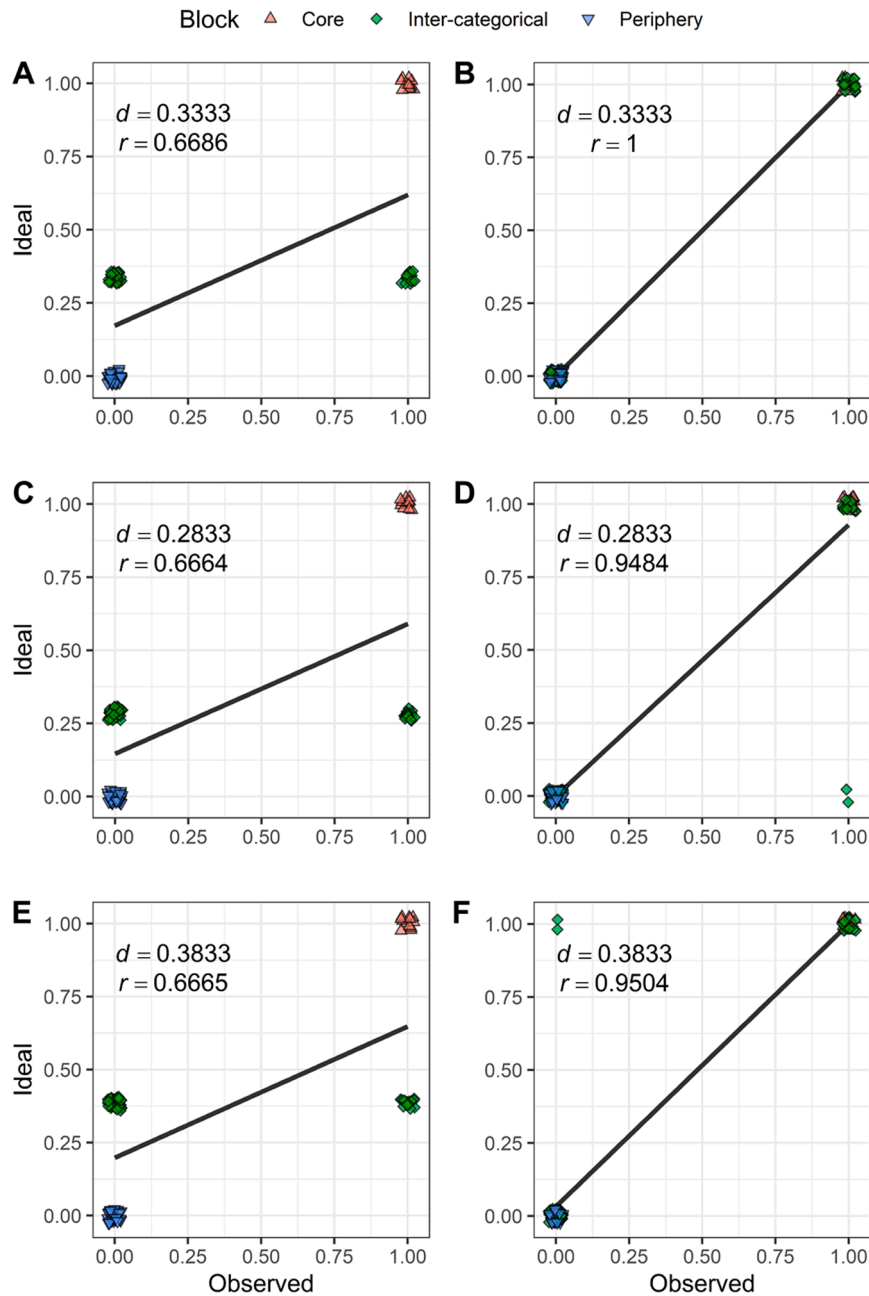


Fig. 3. Impact of the inter-categorical pairs on the goodness-of-fit metric. Visualization of the calculation of the goodness-of-fit metric (r) using the standard Borgatti-Everett method (left) compared to the exact-density blocks method (right). The results correspond to the optimal partition of the *BEfig1* example, with different specified inter-categorical block densities (d): 0.3333 (top), 0.2833 (middle), and 0.3833 (bottom). Data points have been jittered to improve visibility.

the original approach yields slightly higher r -values for d -values of 0.3, 0.35, 0.4, 0.45, and 0.5, even though the retrieved partition remains the same. The same pattern occurs in the *Baker* network. Only in the *Zachary* example, where the exact-density blocks approach with $d = 0.20$ leads to a different partition, this pattern does not hold. Overall, this alignment confirms the effectiveness of the exact-density blocks approach in achieving a more accurate fit to the specified criteria.

Minimum-density ideal blocks

While the exact-density blocks approach effectively resolves the issues associated with specifying non-integer inter-categorical block densities, its practical utility is limited, as researchers often lack a clear estimate of the desired inter-categorical block densities. This makes the

choice of d somewhat arbitrary and, as a result, the standard practice has been to sidestep this issue entirely by setting d to "missing data," a solution that has become the default approach in using the Borgatti and Everett's method.

This "setting-to-missing" solution is undeniably practical and straightforward, as [Boyd et al. \(2006\)](#) noted. However, it fully disregards inter-categorical ties when partitioning the network. We contend that this omission is not only suboptimal but also potentially problematic, as core-periphery structures inherently imply a meaningful level of connectivity between core and periphery members ([Rombach et al., 2017](#)). Moreover, providing users with some control over this connectivity seems to be a desirable feature.

To address these limitations, we propose an alternative use of density blocks that retains the flexibility of the "setting-to-missing" approach

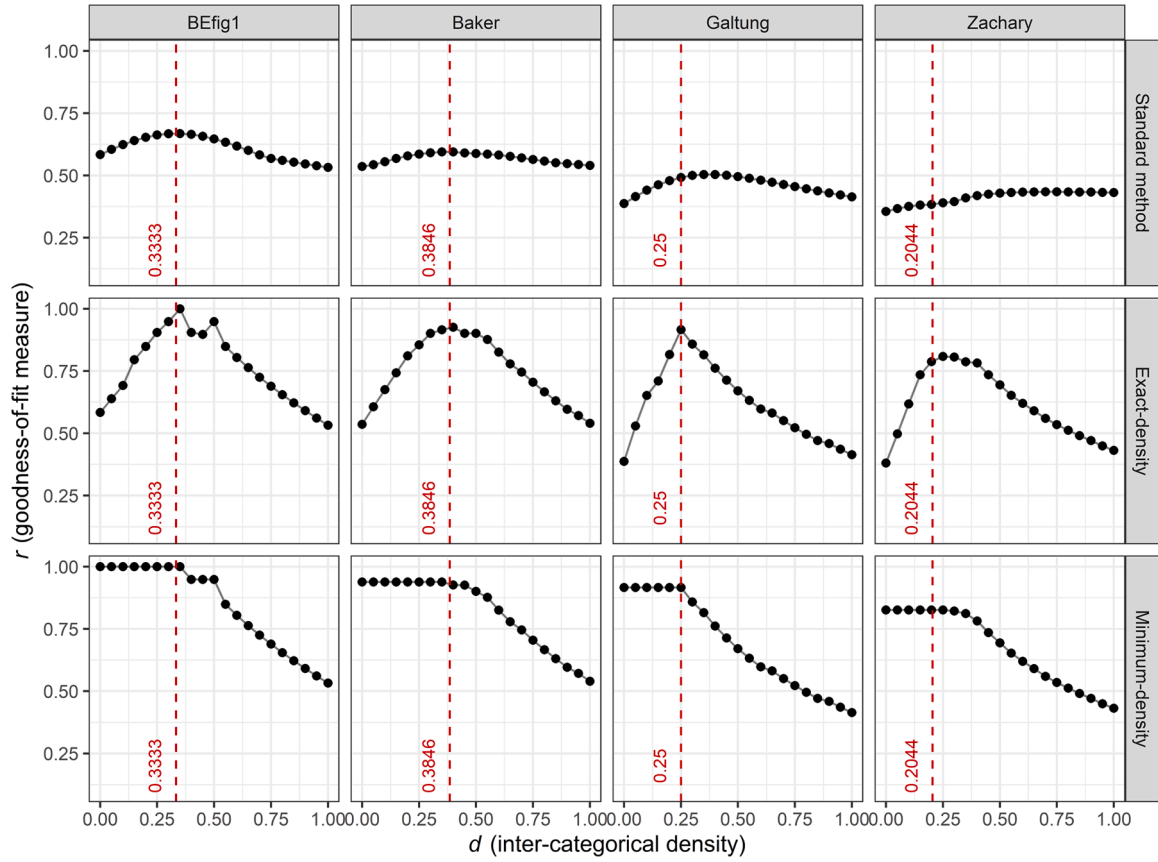


Fig. 4. Goodness-of-fit by inter-categorical density sought. Top panels show results using the standard Borgatti-Everett approach. Middle panels show results with the exact-density block approach. The bottom panels show results with the minimum-density block approach. Red dashed lines represent the actual inter-categorical densities observed in the partition retrieved after setting $d = \text{NA}$, where information about inter-categorical ties is ignored.

while introducing the ability to penalize partitions where the core and periphery are insufficiently connected. This alternative redefines d not as the target density to achieve but as the minimum acceptable level of connectivity between core and periphery members. In the block-modeling literature, this is referred to as using *minimum-density* rather than *exact-density* ideal blocks (cf. Žiberna, 2013, p. 103). The minimum-density approach penalizes partitions that fail to meet the specified minimum density, while leaving partitions that meet or exceed the threshold unpenalized (Doreian et al., 2005, pp. 215, 224).

The implementation of minimum-density ideal blocks closely resembles that of exact-density blocks. First, the inter-categorical blocks $\mathbf{A}^{\text{cp}}$ and $\mathbf{A}^{\text{pc}}$ are vectorized and sorted in descending order to form $\mathbf{a}^{\text{cpl}}$ and $\mathbf{a}^{\text{pcl}}$. These sorted vectors are then matched with ideal-pattern vectors $\mathbf{b}(\mathbf{v})^{\text{cp}}$ and $\mathbf{b}(\mathbf{v})^{\text{pc}}$ where:

$$\mathbf{b}(\mathbf{v})_i^{\text{cp}} = \begin{cases} 1 & \text{if } i \leq \lceil d \cdot |\mathbf{A}^{\text{cp}}| \rceil, \\ \mathbf{a}_i^{\text{cpl}} & \text{otherwise,} \end{cases}$$

$$\mathbf{b}(\mathbf{v})_i^{\text{pc}} = \begin{cases} 1 & \text{if } i \leq \lceil d \cdot |\mathbf{A}^{\text{pc}}| \rceil, \\ \mathbf{a}_i^{\text{pcl}} & \text{otherwise.} \end{cases}$$

Since the ideal-pattern representations mirror the values in the sorted vectors $\mathbf{a}^{\text{cpl}}$ and $\mathbf{a}^{\text{pcl}}$, $\mathbf{b}(\mathbf{v})^{\text{cp}}$ is not necessarily identical to $\mathbf{b}(\mathbf{v})^{\text{pc}}$ unless the network is undirected, or all ties are symmetrical. Additionally, while rounding up to the nearest integer suffices for exact-density blocks, rounding up is crucial for minimum-density blocks. Failing to do so may result in misclassification, where a block that slightly misses the minimum-density requirement is incorrectly classified as meeting it. Applying the ceiling function ($\lceil x \rceil$) ensures that any density falling below the specified threshold incurs penalties, thus preserving the

integrity of the evaluation process.⁷

The main advantage of using minimum-density over exact-density ideal blocks lies in its ability to prioritize solutions that meet or exceed the specified inter-categorical density d . High d values impose stricter criteria, requiring solutions to ensure not only a fully cohesive core and an empty periphery but also strong connectivity between the two groups. Conversely, lower d values allow for more leniency in the level of connectivity needed between core and periphery members. This distinction between exact- and minimum-density blocks becomes evident when examining the middle and bottom panels in Fig. 4, where the r -values of the retrieved partitions are plotted against the pre-specified d values.

At the extreme, setting $d = 0$ prompts the model to select the partition with the highest r -value, irrespective of the observed inter-categorical connectivity. In such cases, the minimum-density blocks approach behaves similarly to setting $d = \text{NA}$. For instance, applying the minimum-density block approach with $d = 0$ to *BEfig1*, *Baker*, and *Galtung* produces the same partitions as those shown in Fig. 2 (with $d = \text{NA}$), only that with r -values of 1.000, 0.9383, and 0.9165, respectively.

Such a similarity notwithstanding, because correlating inter-categorical values with themselves impacts both the goodness-of-fit

⁷ Notice that correlating remaining cell values with themselves is intuitive for binary networks, where we know the maximum observed value (i.e., 1). However, this method poses challenges for valued networks with non-standardized edge values. Thus, before conducting a core-periphery partition for a valued network that involves minimum-density blocks, it is advisable to linearly rescale the values, transforming the valued networks into a standardized range [0,1]. The Socnet.se software client provides this functionality through the *rescale()* command.

measurement r and the mean values $\bar{a}$ and $\bar{b}(\mathbf{v})$ in the calculation of r , the optimal partition retrieved using this approach does not necessarily coincide with that obtained through the "setting-to-missing" solution. While both inter-categorical approaches accommodate *any* inter-categorical content, the minimum-density blocks with $d = 0$ introduces a key difference: the static density of the *entire* network is used to penalize deviations in intra-categorical densities (i.e., missing intra-core ties and unwanted intra-peripheral ties). This contrast with the "setting-to-missing" approach, which relies on the variable combined densities (i.e. mean values) of the intra-core and intra-peripheral blocks. Consequently, although the specific densities of the inter-categorical blocks are irrelevant under the minimum-density block approach with $d = 0$, incorporating the empirical densities of potential inter-categorical ties translates into a more consistent method for penalizing intra-categorical deviations across tested partitions.

This distinction between the two inter-categorical alternatives becomes clear when applying both approaches to more complex networks than *BEfig1*, *Baker*, and *Galtung*. For example, in the *Zachary* network, employing a minimum-density blocks approach with $d = 0$ produces a different partition. In this partition, seven actors (1, 2, 3, 4, 32, 33, and 34) are assigned to the core, as opposed to nine in the "setting-to-missing" solution, yielding an r value of 0.8262. To further compare these two approaches ($d = \text{NA}$ vs minimum-density blocks with $d = 0$), we proceed by applying them to synthetic networks.

Comparing minimum-density blocks and the standard Borgatti-Everett implementation

Given that these two methods (the standard Borgatti-Everett implementation with $d = \text{NA}$ and the minimum-density blocks approach with $d = 0$) can yield different partitions, a critical question arises: does the latter offer a viable alternative, or does one approach demonstrate better performance over the other? To explore this, we conducted experiments to compare the accuracy of both approaches in classifying actors as either core or periphery. These analyses required prior knowledge of the correct partition, so we used synthetic networks based on stochastic blockmodel images for evaluation.

We developed a family of synthetic networks based on a 4-block model, defined by four parameters: $CP(N, c, p, k)$. Here, N represents the number of actors in the network, c is the number of core actors (with $N - c$ as the number of periphery actors), $p \in [0, 1]$ is the probability of a periphery-periphery tie, and $k \in [1, p^{-1/2}]$ scales the probability of inter-categorical and core-core ties. Specifically, the probabilities are defined as follows:

$$\begin{pmatrix} k^2p & kp \\ kp & p \end{pmatrix}.$$

When $k = 1$, the network is an Erdős-Rényi graph, where the probability of all ties is p . As k approaches $p^{-1/2}$, the network increasingly takes on the characteristics of a core-periphery structure, with inter-categorical ties being k times more likely than periphery-periphery ties, and core-core ties k^2 times more likely. For high values of k , the resulting network forms a densely interconnected core, while peripheral actors are preferentially attached to core members but remain sparsely connected among themselves—consistent with the expected properties of a core-periphery structure.

In our experiment, we fixed N to 50 and $p = 1/9$. We then generated 50 networks for each combination of $c = 5, 10, 15, \dots, 45$, and $k = 1, 1.1, 1.2, \dots, 3$ for a total of $50 \cdot 9 \cdot 21 = 9,450$ networks. Each core-periphery approach ($d = \text{NA}$ and minimum-density blocks with $d = 0$)

was applied to these synthetic networks, and the resulting partitions were compared to the predefined ones to assess classification accuracy. Specifically, we determined whether a core actor was correctly classified as "true core" or misclassified as "false periphery", and similarly for peripheral actors as "false core" or "true periphery".

Fig. 5 presents the results. In the top two panels, each vertical bar represents the averaged confusion matrix (comprising "true core", "false core", "true periphery", and "false periphery") across the 50 network versions with the same parameters c and k . To rigorously compare the accuracy of both approaches, we further applied the following regression model:

$$\begin{aligned} \text{logit}(\text{Pr}(\text{success}_i = 50)) = & \beta_0 + \beta_1 \text{approach}_i + \beta_2 k_i + \sum_{j=1}^8 \beta_{2+j} c_i + \beta_{11} (\text{approach}_i \\ & \times k_i) + \sum_{j=1}^8 \beta_{11+j} (\text{approach}_i \times c_i) + \sum_{j=1}^8 \beta_{19+j} (k_i \\ & \times c_i) + \sum_{j=1}^8 \beta_{27+j} (\text{approach}_i \times k_i \times c_i) + u_{\text{network}} \end{aligned}$$

In this model, we estimate the probability of correctly classifying all 50 actors in a given network based on the approach used ($d = \text{NA}$ vs minimum-density blocks with $d = 0$), the value of k , and a series of dummy variables representing different core sizes (c_i , where $i \in \{10, 15, 20, 25, 30, 35, 40, 45\}$). The model also incorporates all two-way and three-way interactions among these predictors. The random-effects term u_{network} accounts for the paired nature of our data, since for each network instance we obtained two classifications depending on the approach used. u_{network} is assumed to follow a normal distribution with a mean of 0 and a variance that is estimated by the model ($\sigma^2_{u_{\text{network}}}$). To further evaluate the comparison, we plotted the predicted probabilities along with their 99.5% confidence intervals in the bottom panels of Fig. 5.

As expected, higher values of k (shown on the x-axis) correspond to more accurate partitioning, regardless of the size of the core and the approach used. However, notable differences arise between the two approaches. When c is small, the minimum-density approach with $d = 0$ proves to be more precise than the standard solution with $d = \text{NA}$. As the sizes of the core and periphery become more similar, the "setting-to-missing" approach tends to perform slightly better. When the core size continues to increase, the "setting-to-missing" solution works better at lower k values, but the minimum-density blocks approach often regains its superiority as k rises.

Overall, the standard method appears more prone to misclassifying peripheral actors as core members (indicated by the red segments of the bars), whereas the minimum-density approach is more likely to misclassify core actors as periphery members (indicated by the brown segments of the bars). This makes the minimum-density blocks approach with $d = 0$ a viable alternative particularly in contexts where the core is expected to be small, as it minimizes the risk of overestimating the size of the core.

Further validation comes from additional tests conducted on synthetic networks with alternative stochastic blockmodels—for instance, models where the inter-categorical tie probability equals k^2p instead of kp , or networks featuring more than a single core-periphery pair. These tests reinforce the conclusion that the minimum-density blocks approach with $d = 0$ is preferable in contexts with smaller cores, regardless of the network's structural characteristics, such as the nature of connectivity between the core and periphery (whether sparse or dense) or the potential presence of multiple cores (see Appendix).

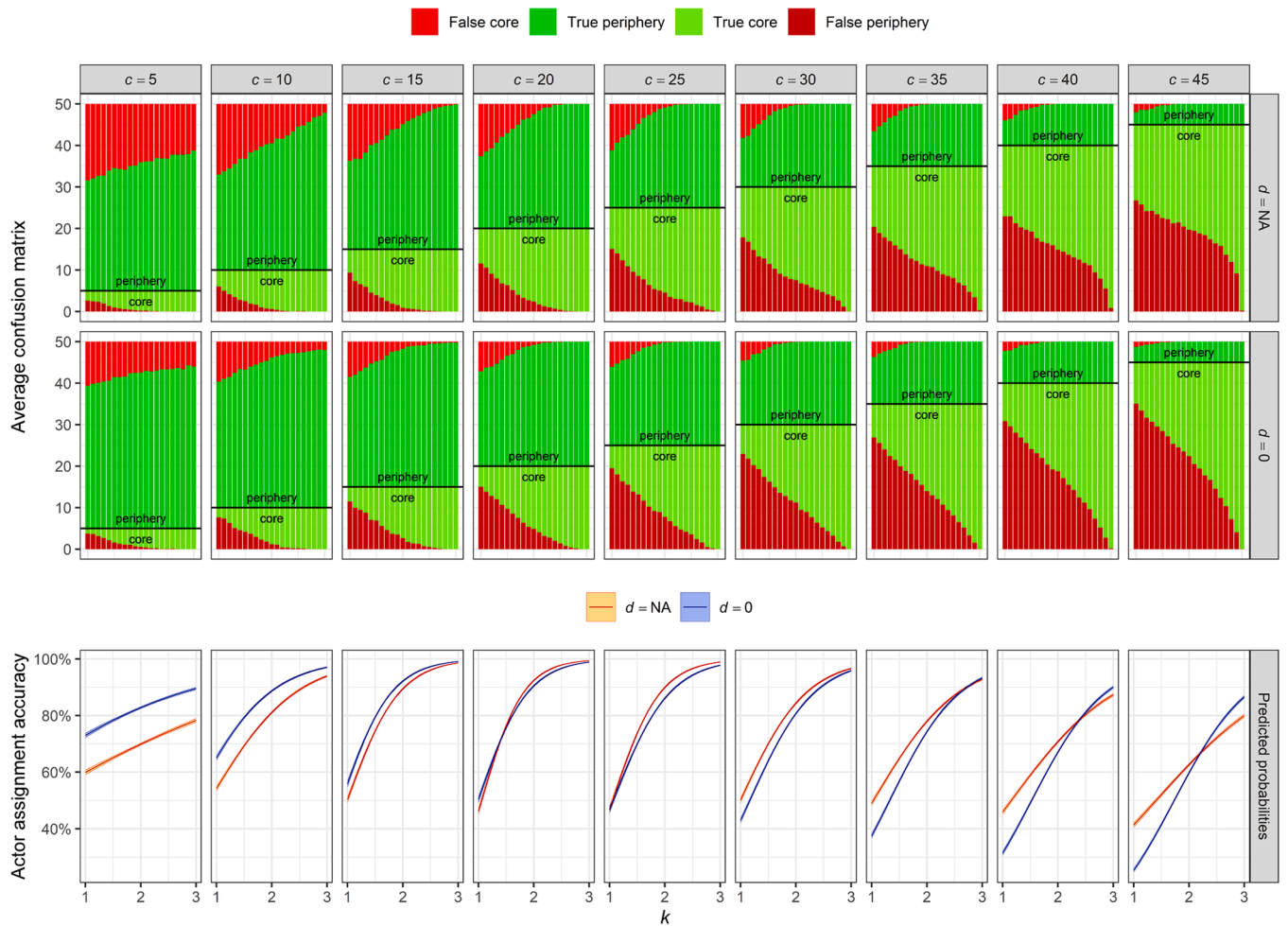


Fig. 5. Performance comparison in core-periphery assignment by core size and approach employed. Top panels show the average confusion matrices for 50 synthetic networks sharing the same k and c parameters, employing the standard “setting-to-missing” approach. Middle panels show the average confusion matrices for the same networks using the minimum-density blocks approach with $d = 0$. Bottom panels illustrate the predicted probabilities of correctly classifying all 50 actors within each network. The orange line represents the results obtained using the “setting-to-missing” approach, while the blue line indicates the probabilities from the minimum-density blocks approach with $d = 0$.

On the full connectedness of ideal-pattern cores

In examining the Borgatti-Everett model, readers may observe a notable discrepancy between *Galtung’s* ideal-typical network topology—considered a classic core-periphery structure (Nordlund, 2018)—and its corresponding core-periphery score. This disparity is largely due to the model’s stringent operationalization of a core block, evaluated against a fully connected block where all possible ties between core members are present. While this method is rigorous, it becomes increasingly impractical as the size of the core grows, since not all connections are likely to be realized. Additionally, this definition might be overly restrictive, failing to capture the broader nature of core-periphery structures, which typically involve a densely connected core but not necessarily a fully connected one.

In a companion piece (Everett and Borgatti, 2000), the authors acknowledged this limitation by introducing the concepts of k -plexes and k -core as alternatives to delineate a network’s core. A k -plex of size n requires that each actor in the core be connected to at least n minus k other core actors (Seidman and Foster, 1978, p. 142; also Wasserman

and Faust, 1994, p. 265). The k -core, on the other hand, specifies the minimum number of core-to-core ties each actor must possess (Seidman, 1983; also Wasserman and Faust, 1994, p. 266ff). Despite the early recognition of this issue, it is surprising that alternative representations of an ideal core—whether k -core, k -plex, or some proportional variant—have not been more fully integrated into the Borgatti-Everett approach.

The p -core ideal block implementation

To overcome this limitation, we adapted some of the tools developed for modifying the treatment of inter-categorical blocks, allowing for more flexible core membership criteria. This new routine, termed the p -core, uses a parameter p (where $p \in [0, 1]$) to define the minimum proportion of other core members to which each actor in the core must be directly connected. Since this method only alters the treatment of values within the core block (A^{cc}), it can be applied alongside any of the previous implementations, including $d = NA$, which governs how ties falling in blocks A^{cp} and A^{pc} are handled.

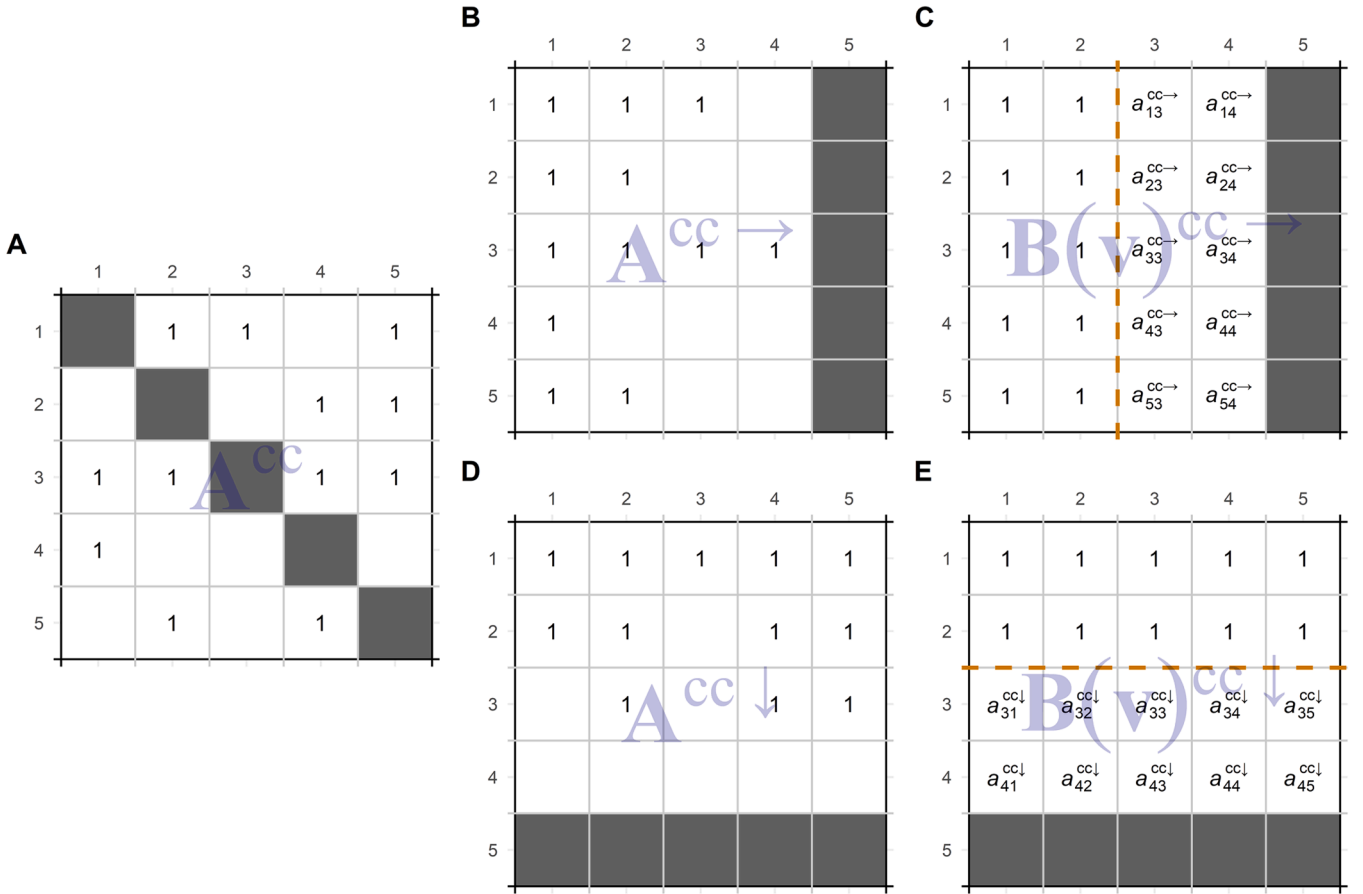


Fig. 6. Visualization of the p -core routine. (A) Core block. (B) Core block with values sorted in decreasing order by row. (C) Ideal-pattern presentation for B with $p = 0.5$. (D) Core block with values sorted in decreasing order by column. (E) Ideal-pattern presentation for D with $p = 0.5$.

The implementation of the p -core extension consists of three main steps (here specified as applicable to both directional and symmetric networks). Additionally, as with the minimum-density blocks method (see footnote 5), it is advisable for valued networks to linearly rescale tie strengths, transforming them into a standardized range $[0,1]$.

1. The core block A^{cc} is sorted row-wise in descending order $A^{cc\rightarrow}$, and matched with an ideal-pattern block $B(v)^{cc\rightarrow}$ where:

$$b(v)_{ij}^{cc\rightarrow} = \begin{cases} 1 & \text{if } i \leq \lceil p \cdot (|A^{cc}|_{\text{rows}} - 1) \rceil \\ a_{ij}^{cc\rightarrow} & \text{otherwise.} \end{cases}$$

Here, $|A^{cc}|_{\text{rows}}$ represents the number of rows in the core block (or columns, since A^{cc} is a square matrix), and the " -1 " accounts for missing diagonal values. The ceiling function ensures appropriate rounding.

2. In this second step, the core block is sorted column-wise in descending order $A^{cc\downarrow}$, and matched with another ideal-pattern block $B(v)^{cc\downarrow}$, where:

$$b(v)_{ij}^{cc\downarrow} = \begin{cases} 1 & \text{if } j \leq \lceil p \cdot (|A^{cc}|_{\text{rows}} - 1) \rceil \\ a_{ij}^{cc\downarrow} & \text{otherwise.} \end{cases}$$

Fig. 6 illustrates both steps for the network partition in Fig. 1 with

$p = 0.5$.

3. Finally, values from both core block images are incorporated into the goodness-of-fit metric, along with values from other blocks. Since the p -core routine doubles the observations from the core block, the standard goodness-of-fit metric (r) is replaced with a weighted version to account for the proportional influence of core observations (Nordlund, 2020). The weighted goodness-of-fit metric r_w is given by:

$$r_w = \frac{\sum w_{ij} (a_{ij} - \bar{a}) (b_{ij}(v) - \bar{b}(v))}{\sqrt{\sum w_{ij} (a_{ij} - \bar{a})^2 \sum w_{ij} (b_{ij}(v) - \bar{b}(v))^2}}$$

where $w_{ij} = 0.5$ for core block observations and $w_{ij} = 1$ for all other observations.

Similar to the minimum-density block solution, the p -core routine ensures that partitions where core actors lack sufficient ties to other core members are penalized. These weak partitions contribute (0,1) pairs to the goodness-of-fit measure, while those with all core actors meeting the necessary tie threshold (as determined by p) contribute only (1,1) and (0,0) pairs.

Notice that, for undirected networks (as those we analyze here), the p -core implementation can be simplified, as steps 1 and 2 will necessarily yield identical results. However, this is not the case for directed networks. Since our goal was to create a version adaptable to directed networks as well, we designed it as a three-step procedure to

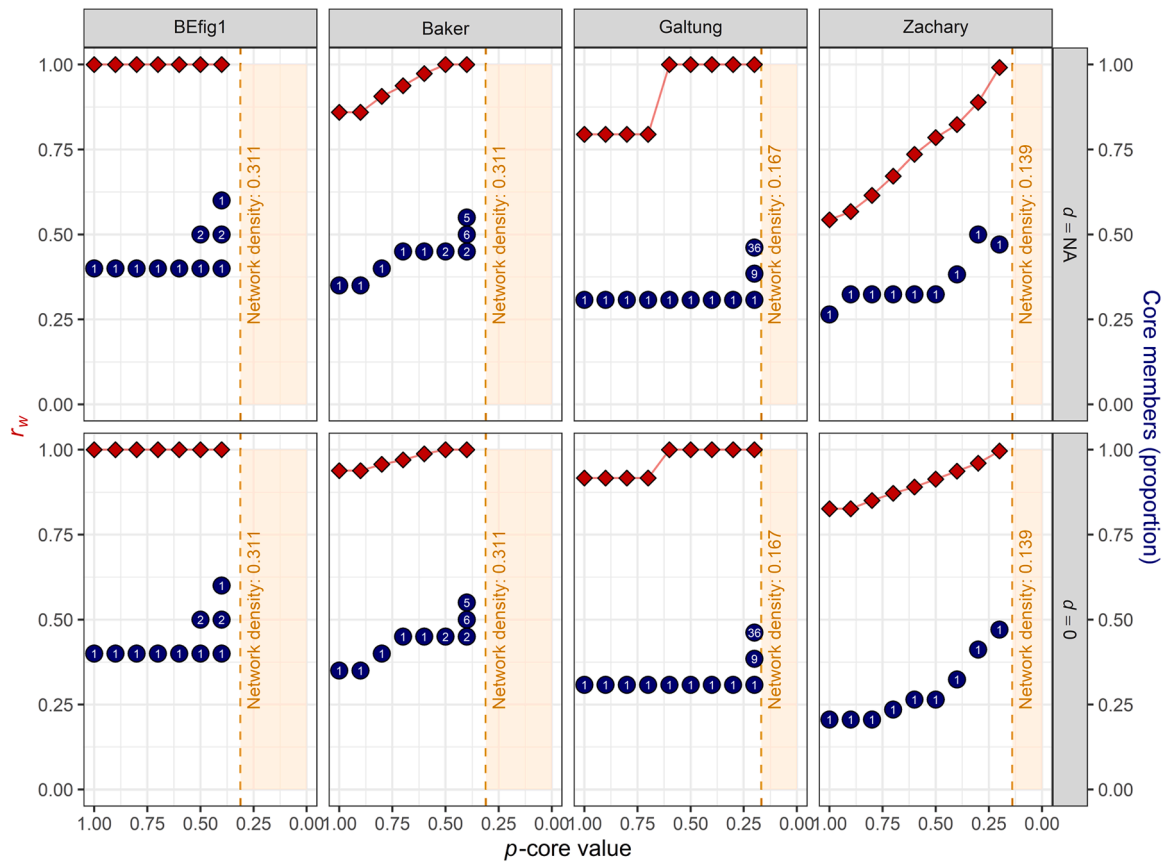


Fig. 7. Impact of the p -core value on the partition and goodness-of-fit measure. Top panels show results using Borgatti and Everett's standard treatment of inter-categorical ties ($d = NA$). Bottom panels show results using the minimum-density blocks approach with $d = 0$. Red rhombuses indicate the goodness-of-fit measure values, while the numbers inside the blue dots represent the number of unique partitions with the same number of core members.

accommodate both cases effectively.

The p -core routine in practice

Fig. 7 displays results from the p -core extension applied to the *BEfig1*, *Baker*, *Galtung*, and *Zachary* networks. For each network, we extracted the r_w values (indicated by red rhombuses) across various partitions for different values of p (1, 0.9, 0.8, ..., down to the network's overall density). The y-axis represents the proportion of core actors, with numbers inside the blue dots indicating the number of unique partitions with the same number of core members. Top panels show the resulting partition using Borgatti and Everett's standard treatment of inter-categorical ties (with $d = NA$), while bottom panels use the minimum-density blocks implementation with $d = 0$.

Two key observations arise from these results. First, as p decreases, it becomes more likely that actors initially classified as peripheral will shift into the core. In *BEfig1*, for instance, actors 5 and 8 (both sharing ties with two core members) are reclassified as core members when p is 0.5 or lower. Second, as p approaches zero, r_w tends to 1, as the p -core conquers the network. Thus, a careful selection of p is recommended, with consideration of overall edge density (indicated by orange in Fig. 7).

The p -core implementation offers two advantages. One is its ability to reveal how network partitions shift or remain stable as the criterion for core membership becomes more flexible. Stable partitions across a broad range of p values indicate a robust core-periphery division, with clear boundaries between the core and the periphery. In contrast, variability in partitions as p changes suggests a more complex network structure, possibly indicating a pseudo-periphery or ridge structure—areas with varying connectivity levels (Friedkin, 1998)—or the

presence of multiple core-periphery pairs within the network (Kojaku and Masuda, 2017; Rombach et al., 2017). For example, in the *Galtung* network, a clear core-periphery separation is observed. In *BEfig1*, actors 5 and 8 form a grey area that becomes noticeable when p is 0.5 or lower. The *Baker* and *Zachary* networks on their side display a more nuanced division.

A second advantage, aligned with the core objective of this extension, is the p -core routine's capacity to identify core members in situations where missing intra-core ties are plausible or even expected, such as when the core is relatively large. To demonstrate this, we reanalyzed our 9450 synthetic networks, comparing the standard Borgatti and Everett partitions with those obtained using the p -core extension. For consistency, d was set to missing in both scenarios.⁸ For illustrative purposes, we set the p -core value to 0.5, meaning that for a core of size 5, $\lceil 0.5 \cdot (5 - 1) \rceil = 2$ ties to other core members would suffice to avoid a penalty, whereas for a core of size 45, $\lceil 0.5 \cdot (45 - 1) \rceil = 22$ ties would be required. This represents a significant relaxation of the criterion for core membership.

Fig. 8 presents the results. As shown, in networks with a large core, the p -core routine significantly improves actor assignment accuracy by correctly identifying more actors as core members. This improvement is noticeable in networks with large cores, where the enhancement in actor assignment accuracy can exceed 10 %, especially in networks characterized by a well-defined core-periphery structure (indicated by high k values). However, in networks with a small core, the use of the p -core

⁸ The observed patterns remain consistent regardless of whether the standard “setting-to-missing” solution or the minimum-density blocks approach with $d = 0$ is applied.

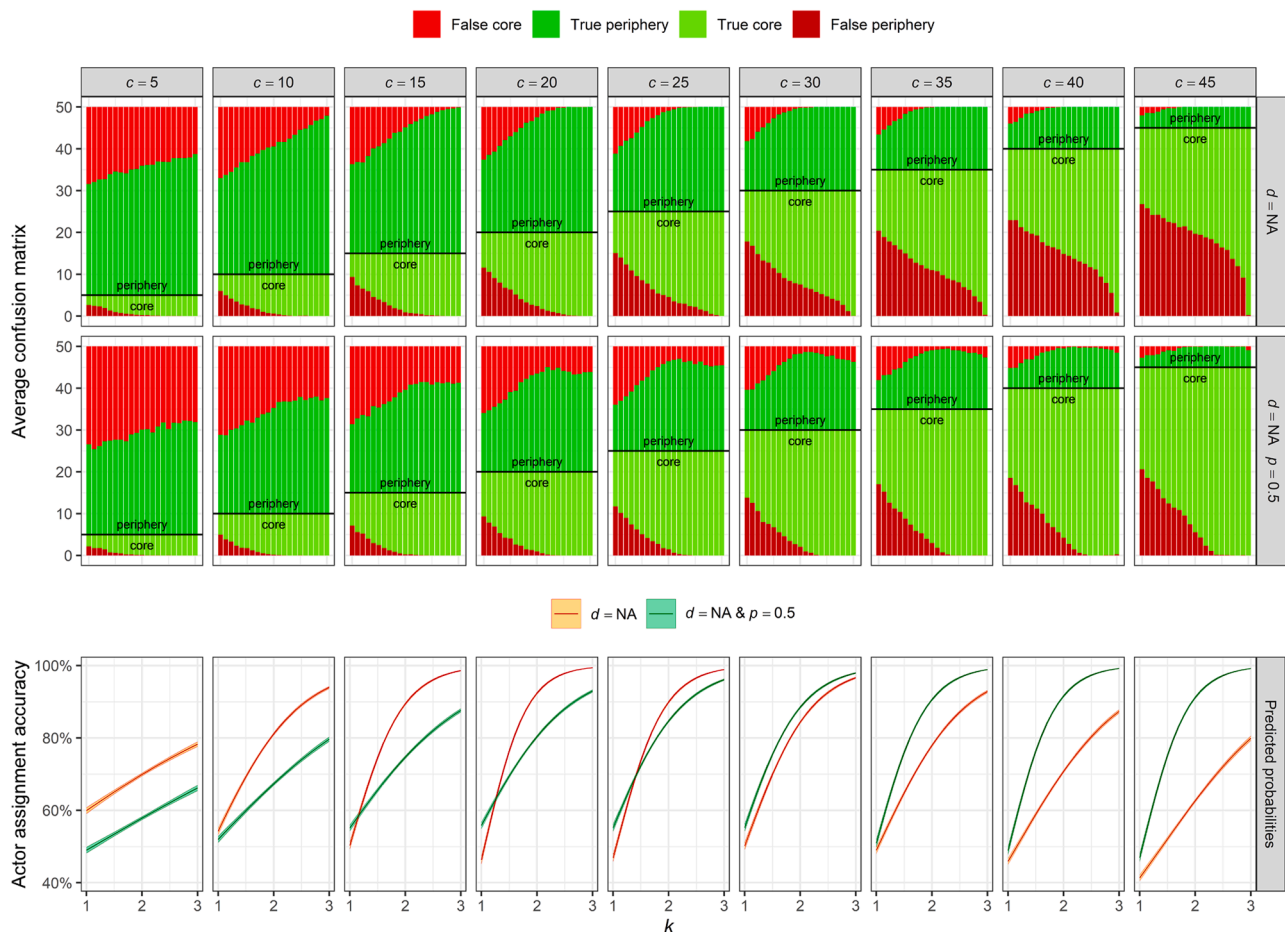


Fig. 8. Performance comparison in core-periphery assignment by core size and approach employed. Top panels show the average confusion matrices for 50 synthetic networks sharing the same k and c parameters, employing the standard “setting-to-missing” approach. Middle panels show the average confusion matrices for the same networks using the p -core extension with $p = 0.5$. Bottom panels illustrate the predicted probabilities of correctly classifying all 50 actors within each network. The orange line represents the results obtained using the “setting-to-missing” approach, while the green line indicates the probabilities using the p -core extension with $p = 0.5$.

has the opposite effect. Since this extension relaxes the criteria for classification as a core member, it increases the likelihood of misclassifying peripheral members as core actors in such scenarios.

Similar to the minimum-density blocks approach with $d = 0$, we tested the performance of the p -core extension on synthetic networks using alternative stochastic blockmodels (see Appendix). The results underscore the high versatility of the p -core extension across diverse network structures. In particular, it demonstrates accuracy improvements in core-periphery structures with dense connectivity between the core and its periphery, provided the core is relatively large. Moreover, the p -core extension proves effective in networks featuring multiple cores, regardless of the connectivity type between these cores—whether completely disconnected, directly connected, or linked through their respective peripheries.

To further evaluate the compatibility of our two extensions, we also combined the p -core extension with the minimum-density blocks approach to manage inter-categorical ties. The results confirm that these approaches complement each other, enhancing classification accuracy across a range of network configurations (see Fig. A6 in the Appendix). These findings validate the adaptability and utility of the p -core extension across a wide range of complex network configurations.

Discussion

It has been 25 years since Borgatti and Everett (2000) first operationalized the notion of core-periphery structures and introduced a set of strategies for identifying such patterns in observed data. Since then, numerous alternative approaches have been proposed (e.g. Boyd et al., 2006, 2010; Cucuringu et al., 2016; Della Rossa et al., 2013; Gallagher et al., 2021; Kojaku and Masuda, 2017, 2018; Lip, 2011; Rombach et al., 2014; Tang et al., 2019; van Lidth de Jeude et al., 2019). Despite these advancements, Borgatti and Everett’s method remains appealing due to its simplicity and ease of application. As a result, it is often the go-to tool for researchers mapping network patterns indicative of core-periphery divisions.

This paper sought to further refine Borgatti and Everett’s classic tool, addressing limitations in the original implementation related to specifying density values for inter-categorical blocks, while also enhancing its versatility. Crucially, we ensured that our extensions retain a straightforward interpretation—an advantage compared to the more complex parameters found in other algorithms (e.g., Rombach et al., 2017). Additionally, unlike methods that generate a “coreness” score (Boyd et al., 2010; Della Rossa et al., 2013), our extensions operate on the

discrete version of the method, avoiding the need to establish a threshold value for dichotomizing the network into core and periphery. As a result, our extensions are arguably more backward-compatible with the original Borgatti-Everett model for identifying and evaluating core-periphery structures.

As demonstrated, the use of exact-density blocks ensures that the retrieved partitions are optimized according to a goodness-of-fit metric that accurately reflects the pre-specified block density criteria—a shortcoming in the original method. However, if inter-categorical densities are to be specified, we advocate for the minimum-density block approach as it offers more flexibility. On one hand, the minimum-density approach can closely replicate the most common application of the original method (with $d = \text{NA}$) by setting $d = 0$, which is particularly advantageous when no prior knowledge exists regarding the degree of connectivity between the core and its periphery. Tests conducted on synthetic networks revealed that this approach not only serves as a viable alternative to the standard "setting-to-missing" method but often outperforms it in accurately classifying core members, particularly in networks with small cores. On the other hand, when the connectivity between core and periphery members in the retrieved partition is deemed unsatisfactory (e.g., too low), the researcher can incrementally increase the d -value to search for a more integrated core-periphery structure.

The p -core implementation offers another advantage, helping delineate the core in networks where it is large and some intra-core ties may remain unrealized. This property became evident during analysis of synthetic networks designed with stochasticity in intra-core ties. Moreover, this extension can be used to assess the consistency of core-periphery partitions by relaxing the intra-core connectivity threshold, providing useful comparability across networks regarding the rigidity of their core-periphery structures.

The proposed extensions, while valuable, have limitations similar to those of the original Borgatti and Everett method. Notably, neither the original model nor our extensions are explicitly designed to capture multiple core-periphery pairs within a network. As a result, the resulting partition simply classifies individuals as core or peripheral, without distinguishing between different core groups. Consistent with the broader logic of blockmodeling, our focus remains on categorizing actors based on shared roles, rather than delineating communities. For researchers interested in algorithms that specifically address hybrid core-periphery and community structures, we recommend exploring the approaches proposed by Kojaku and Masuda (2017) or Rombach et al. (2017).

A second limitation is that neither the original method nor our extensions account for the generative mechanisms underlying the networks. Borgatti and Everett themselves acknowledged this limitation, emphasizing the exploratory nature of their method. More recent methods proposed (e.g., Della Rossa et al., 2013; Kojaku and Masuda, 2017; Rombach et al., 2017) incorporate some form of statistical testing, often comparing quality scores against simulated networks (e.g., Erdős-Rényi random networks with the same number of nodes and ties or degree distributions). In line with the philosophy of the original tool, we did not incorporate such testing, leaving the method without a formal mechanism for determining whether the retrieved partition reflects a meaningful core-periphery structure. That said, many of these testing mechanisms can be extended to both the original Borgatti and Everett method and our proposed extensions.

Here, we strictly adhered to the "density-based" approach (Lee et al., 2014) of the original formulation, particularly in managing inter-categorical ties at the block level rather than the individual level. In this method, the relationships between core and peripheral actors are assessed collectively, based on the density of ties within the entire block. For example, if $d \geq 0.25$ is specified in a network consisting of four core actors and eight peripheral actors, the inter-categorical block, which contains $4 \times 8 = 24$ possible ties, would be considered sufficiently connected if 8 ties (25 %) are present. This condition holds regardless of

whether those ties involve all peripheral actors or are concentrated among just a few. In contrast, an individual-level implementation would require that each peripheral actor be directly connected to at least one core member, thus ensuring a more fine-grained analysis of actor-to-actor relationships. To accommodate such analyses, additional implementations that follow this logic have been incorporated into the Socnet.se software client. In these implementations, block images are constructed using row-regular and column-regular blocks (e.g. Doreian et al., 2005, p. 212ff), rather than by vectorizing the entire block. A similar approach is found in Nordlund (2018), where the power-relational feature of peripheral dependency will penalize peripheral actors that not only lack ties to a core actor, but also if they have more than one tie.⁹

Lastly, while the extensions we propose maintain the versatility of the original Borgatti and Everett method, which can be applied to undirected, directed, and even valued networks, the benefits of our enhancements beyond dichotomous networks warrant further exploration. For instance, in valued (aka weighted) networks, where ties are not simply binary but carry significance in terms of strength or capacity, it is yet to be fully tested how well our extensions adapt to such scenarios. Similarly, the Borgatti and Everett model is primarily designed for one-mode networks, where all nodes belong to the same category or actor set. When dealing with bipartite or multi-mode networks—networks where nodes are divided into two or more distinct sets (e.g., actors and events)—a direct application of our method requires projecting these higher-mode networks into their one-mode equivalents. In bipartite networks, this means collapsing the network into a single-mode projection, potentially losing some of the richness of the original structure. For researchers working with bipartite or higher-mode networks, bipartite blockmodeling techniques may offer a more accurate approach. These methods are designed to preserve the unique relationships between different types of actors in the network, and could thus provide more robust insights than a one-mode projection.

Conclusion

As the concept of core-periphery structures continues to broaden its applicability across various fields, it becomes increasingly important to develop tools that not only ensure comparability but also offer some flexibility. With this objective in mind, the present study extends the Borgatti-Everett core-periphery model (2000), which has become the *de facto standard* for identifying categorical core-periphery structures in empirical data (e.g., Rombach et al., 2017, p. 620).

The implementations introduced here remain faithful to the original intent of Borgatti and Everett—specifically, the detection of densely cohesive cores and sparse, loosely connected peripheries. At the same time, these extensions offer researchers greater flexibility in two areas. First, they enable users to set a desired minimum level of connectivity between core and peripheral actors, thereby tailoring the tool to reflect varying degrees of core-periphery interaction. Second, they introduce flexibility in defining core membership, allowing for some leniency in the extent of intra-core connectivity required to classify actors as core members. Together, these modifications enhance the model's versatility, making it better suited for complex, real-world networks where strict dichotomies between core and periphery may not always hold.

Adapting an established tool to extend its capabilities while preserving the simplicity of the original solution is a challenge. Here, we aimed

⁹ The power-relational features proposed in Nordlund (2018) such as peripheral dependency and core dominance, both implemented in the Socnet.se software client, are fully compatible with the p -core block we propose in this article. For example, to search for a core-periphery structure in the Zachary network with peripheral dependency (Nordlund, 2018, p. 354ff) and a p -core with $p = 0.5$, use the following Socnet.se command: `coreperi(zachary,ljubljana,powerrelational=dep,core=pco(0.5))`.

to strike a balance between maintaining comparability with the original Borgatti-Everett model and introducing the flexibility needed to accommodate diverse research contexts. The outcome is a refined tool that enhances researchers' ability to effectively map observed networks while detecting patterns suggestive of a core-periphery structure. This proves particularly valuable in scenarios where a minimum level of connectivity between core and periphery is necessary, or where a fully integrated core may be improbable due to the nature of the network.

CRedit authorship contribution statement

José Luis Estévez: Writing – review & editing, Writing – original draft, Visualization, Validation, Methodology, Formal analysis. **Carl Nordlund:** Writing – review & editing, Writing – original draft, Software, Methodology, Formal analysis, Conceptualization.

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Declaration of Generative AI and AI-assisted technologies in the writing process

During the preparation of this work the authors occasionally used ChatGPT in order to improve readability and language. After using this tool/service, the authors reviewed and edited the content as needed and take full responsibility for the content of the publication.

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Appendix

Experiment 1: Evaluating performance in core-periphery structures with dense inter-categorical blocks

Several authors have operationalized a core-periphery structure as a network characterized by many inter-categorical ties or, in block terms, as three full blocks and one empty block (Kojaku and Masuda, 2017, 2018; Rombach et al., 2017):

$$\begin{pmatrix} \text{full} & \text{full} \\ \text{full} & \text{empty} \end{pmatrix}.$$

To evaluate the performance of the minimum-density blocks approach with $d = 0$ compared to the standard “setting-to-missing” approach in networks exhibiting this structure, we generated a family of synthetic networks based on the following 4-block model:

$$\begin{pmatrix} k^2p & k^2p \\ k^2p & p \end{pmatrix}.$$

Fig. A1 reveals performance trends consistent with those observed in Fig. 5 of the main text. When c , the size of the core, is small, the minimum-density blocks approach demonstrates higher precision. As the core and periphery sizes become more comparable, the “setting-to-missing” approach shows slightly better performance. However, as the core size continues to increase, the “setting-to-missing” approach outperforms the minimum-density blocks approach only at low k values. As k increases, the minimum-density blocks approach regains its advantage, delivering superior results.

To evaluate potential performance benefits of the p -core extension, we also applied this approach with $p = 0.5$ to the new family of synthetic networks and compared the results to those obtained using the standard Borgatti-Everett method, with d set to missing in both cases. These results are presented in Fig. A2. Consistent with the patterns observed in Fig. 8, here too the p -core extension demonstrates a notable advantage in classification accuracy in networks with a large core. However, in networks characterized by a small core or a more balanced size between core and periphery members, the p -core extension reduces classification accuracy, due to the misclassification of peripheral members as core actors. It is important to note that a p -core value of 0.5 represents a significant relaxation of the criterion for core membership, especially in networks with dense connections between core and periphery members.

Experiment 2: Evaluating performance in two-core networks

To assess how the two approaches ($d = \text{NA}$ vs. minimum-density blocks with $d = 0$) handle networks with more than one core, we generated a second set of synthetic networks based on a 16-block model, representing two juxtaposed core-periphery structures in three different configurations:

1. **Unconnected:** In this configuration, the two core-periphery structures are entirely disconnected, and the probability of a tie within each block is defined as follows:

$$\begin{pmatrix} k^2p & kp & 0 & 0 \\ kp & p & 0 & 0 \\ 0 & 0 & k^2p & kp \\ 0 & 0 & kp & p \end{pmatrix},$$

where 0 indicates the probability of a tie in that block is zero.

2. Core-connected: In this version, the two core-periphery structures are connected through their core members. The probability of a tie within each block is defined as follows:

$$\begin{pmatrix} k^2p & kp & \frac{p}{\sqrt{c}} & 0 \\ kp & p & 0 & 0 \\ \frac{p}{\sqrt{c}} & 0 & k^2p & kp \\ 0 & 0 & kp & p \end{pmatrix}.$$

3. Periphery-connected: In this case, the two core-periphery structures are connected through their peripheral members, and the probability of a tie within each block is defined as follows:

$$\begin{pmatrix} k^2p & kp & 0 & 0 \\ kp & p & 0 & \frac{p}{\sqrt{N-c}} \\ 0 & 0 & k^2p & kp \\ 0 & \frac{p}{\sqrt{N-c}} & kp & p \end{pmatrix}.$$

As in the previous experiment, we fixed $N = 50$ and $p = 1/9$. Additionally, we ensured that the two core-periphery structures always had the same size, with each structure consisting of 25 members, and core members equally split between the two structures. For this experiment, we generated 50 networks for each combination of $c = 10, 20, 30, 40$, $k = 1, 1.1, 1.2, \dots, 3$, and connection type (“Unconnected,” “Core-connected,” “Periphery-connected”), resulting in a total of 12,600 networks. Fig. A3 provides a visualization of some of these network configurations.

We repeated the same analyses as in the previous experiment to assess classification accuracy. The results are presented in Fig. A4. In multi-core networks, both approaches exhibit a higher likelihood of misclassification compared to single-core core-periphery structures, even as k approaches 3. When comparing the two approaches, the minimum-density block method with $d = 0$ shows superior performance in networks with smaller cores, whereas the “setting-to-missing” approach tends to perform better in networks with larger cores or a more balanced core-periphery division. Once again, this highlights the strengths of each method depending on the size of the core.

Consistent with the previous experiments, the minimum-density approach shows a lower tendency to misclassify peripheral members as core, but it is more likely to incorrectly assign core members to the periphery. This suggests that the minimum-density approach with $d = 0$ is particularly well-suited for networks with smaller cores, due to its more conservative criterion for determining core membership.

Fig. A5 presents the results assessing the performance advantages of the p -core extension. For illustrative purposes, we set $p = 0.5$ and d to missing, allowing a direct comparison between the p -core extension and the standard application of the Borgatti-Everett method. Once again, improvements in classification accuracy are evident in networks characterized by a large core. However, in networks with a small core, the extension results in poorer performance, again due to the increased misclassification of peripheral members as core actors. This pattern remains consistent regardless of the type of connectivity between the two cores—whether they are completely disconnected, directly connected, or linked via their respective peripheries—further validating the versatility and utility of the p -core extension across diverse network structures.

Finally, to evaluate how the minimum-density blocks approach interacts with the p -core extension, we reanalyzed the synthetic networks featuring two cores with varying forms of connectivity. In these tests, d was set to 0 to prioritize partitions that maximize goodness-of-fit irrespective of inter-categorical density. Similarly, the p -core value was set to 0.5.

Fig. A6 compares these results to the standard method ($d = \text{NA}$), which requires full connectivity amid core members. The findings reveal that these extensions complement each other effectively, yielding higher overall accuracy compared to the standard method in networks with incipient or robust core-periphery structures. These results underscore not only the potential for combining the two approaches but also their advantage in delivering more accurate classifications across diverse network configurations.

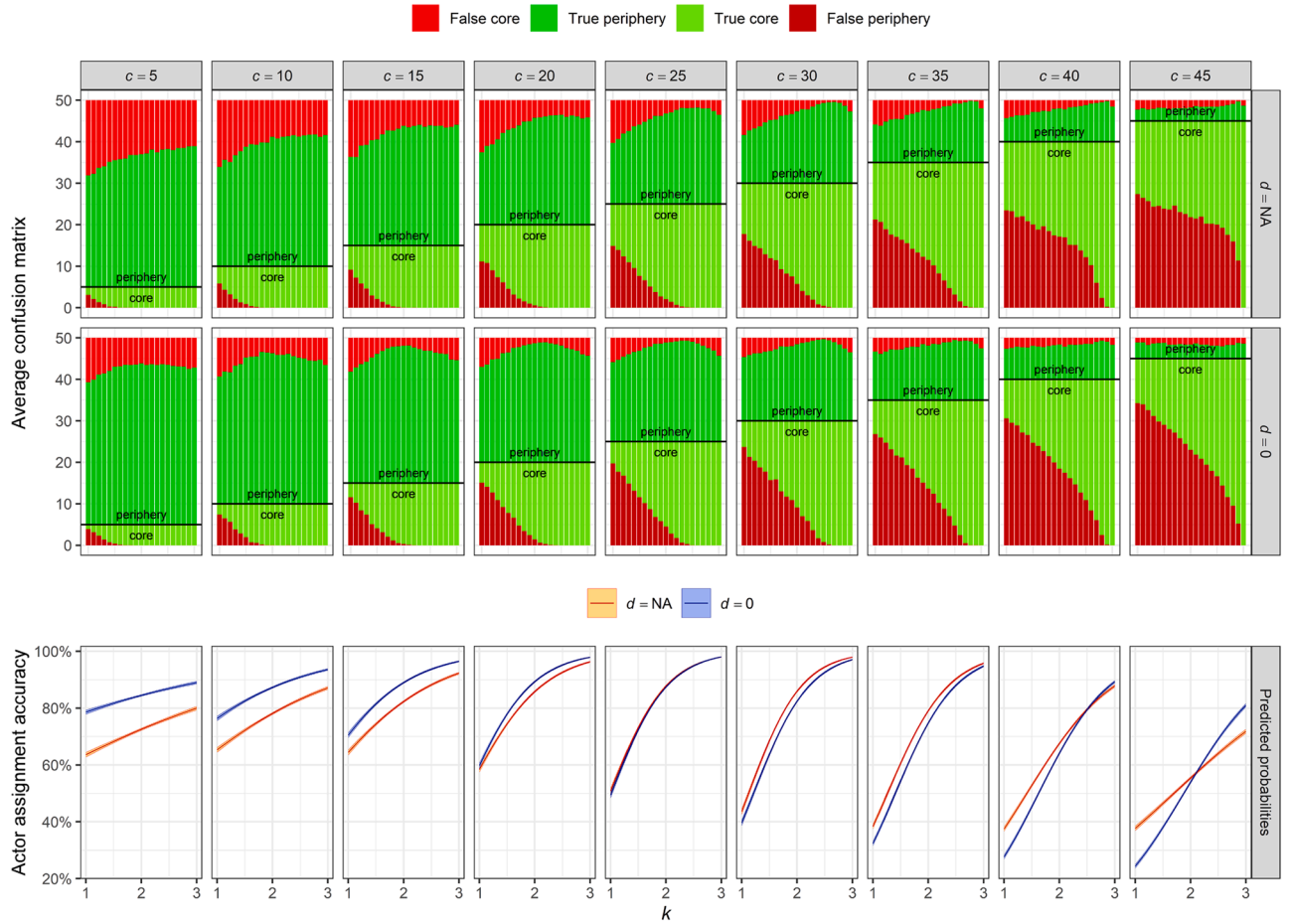


Fig. A1. Performance comparison in core-periphery assignment by core size and approach employed (dense inter-categorical blocks). Top panels show the average confusion matrices for 50 synthetic networks sharing the same k and c parameters, employing the standard “setting-to-missing” approach. Middle panels show the average confusion matrices for the same networks using the minimum-density blocks approach with $d = 0$. Bottom panels illustrate the predicted probabilities of correctly classifying all 50 actors within each network. The orange line represents the results obtained using the “setting-to-missing” approach, while the blue line indicates the probabilities from the minimum-density blocks approach with $d = 0$.

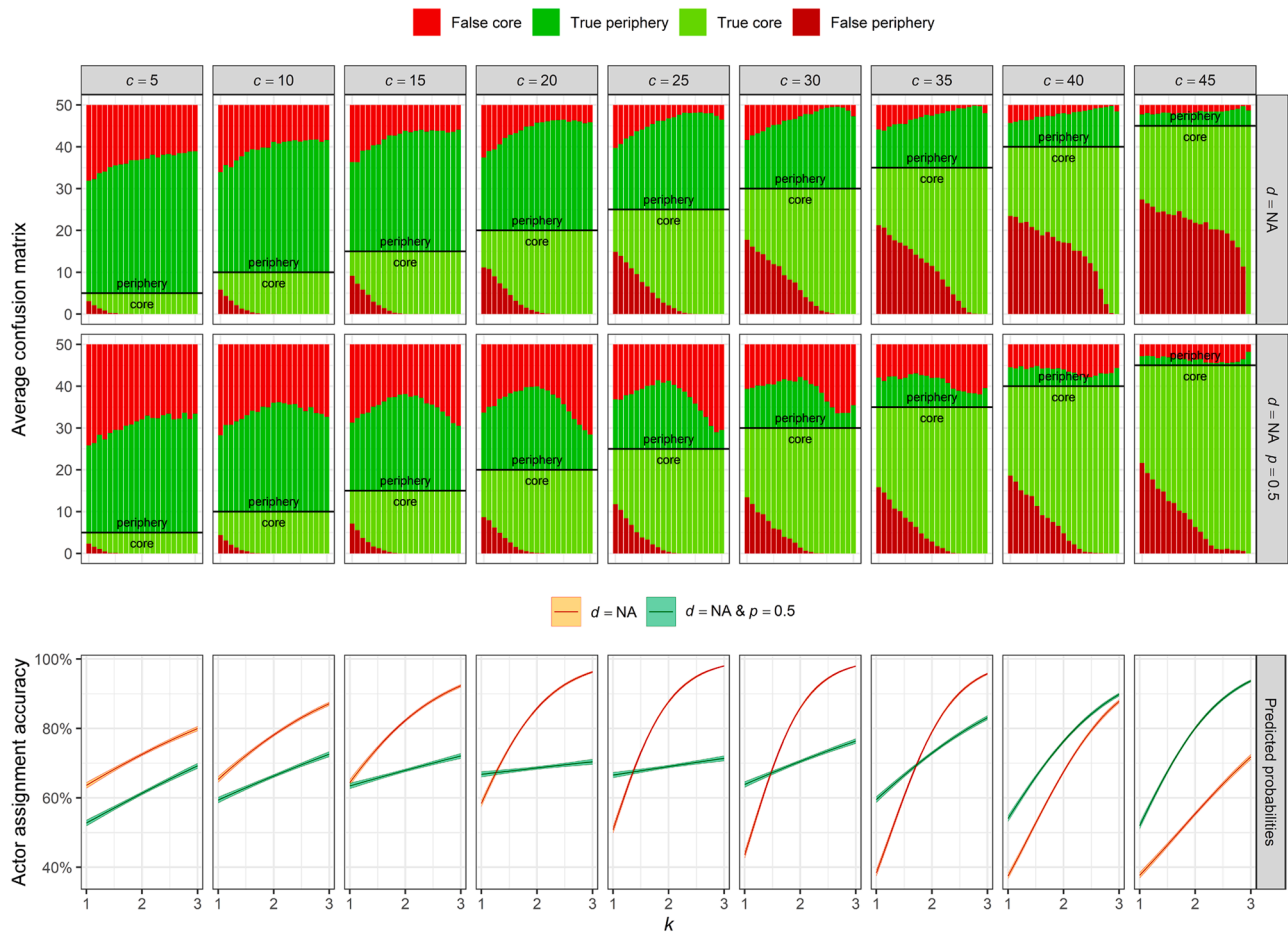


Fig. A2. Performance comparison in core-periphery assignment by core size and approach employed (dense inter-categorical blocks). Top panels show the average confusion matrices for 50 synthetic networks sharing the same k and c parameters, employing the standard “setting-to-missing” approach. Middle panels show the average confusion matrices for the same networks using the p -core extension with $p = 0.5$. Bottom panels illustrate the predicted probabilities of correctly classifying all 50 actors within each network. The orange line represents the results obtained using the “setting-to-missing” approach, while the green line indicates the probabilities using the p -core extension with $p = 0.5$.

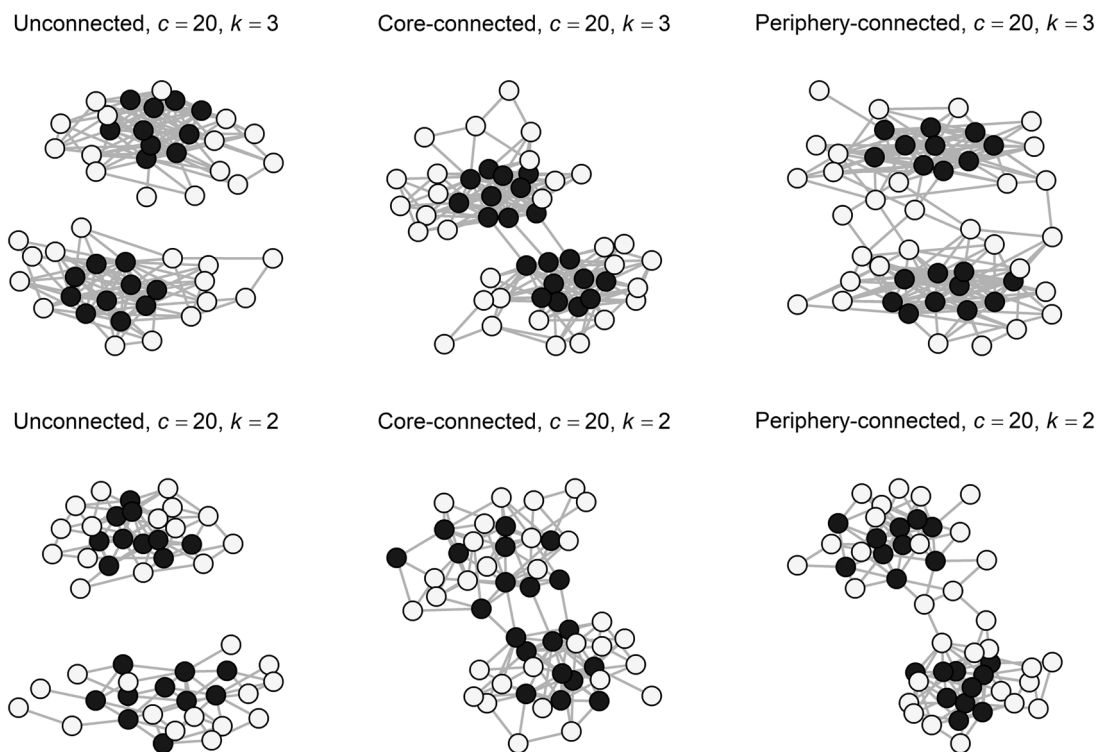


Fig. A3. Examples of two-core synthetic networks. Examples of synthetic networks with a two core-periphery structure under different connection scenarios: Two core-periphery structures with no connections between them (left). The two core-periphery structures are linked through their core members (middle). The two core-periphery structures are connected via their peripheral members (right). The top panels feature networks with a clear core-periphery structure ($k = 3$), while the bottom panels show networks with a more noisy core-periphery structure ($k = 2$). Black circles represent core nodes, and light grey circles represent peripheral nodes.

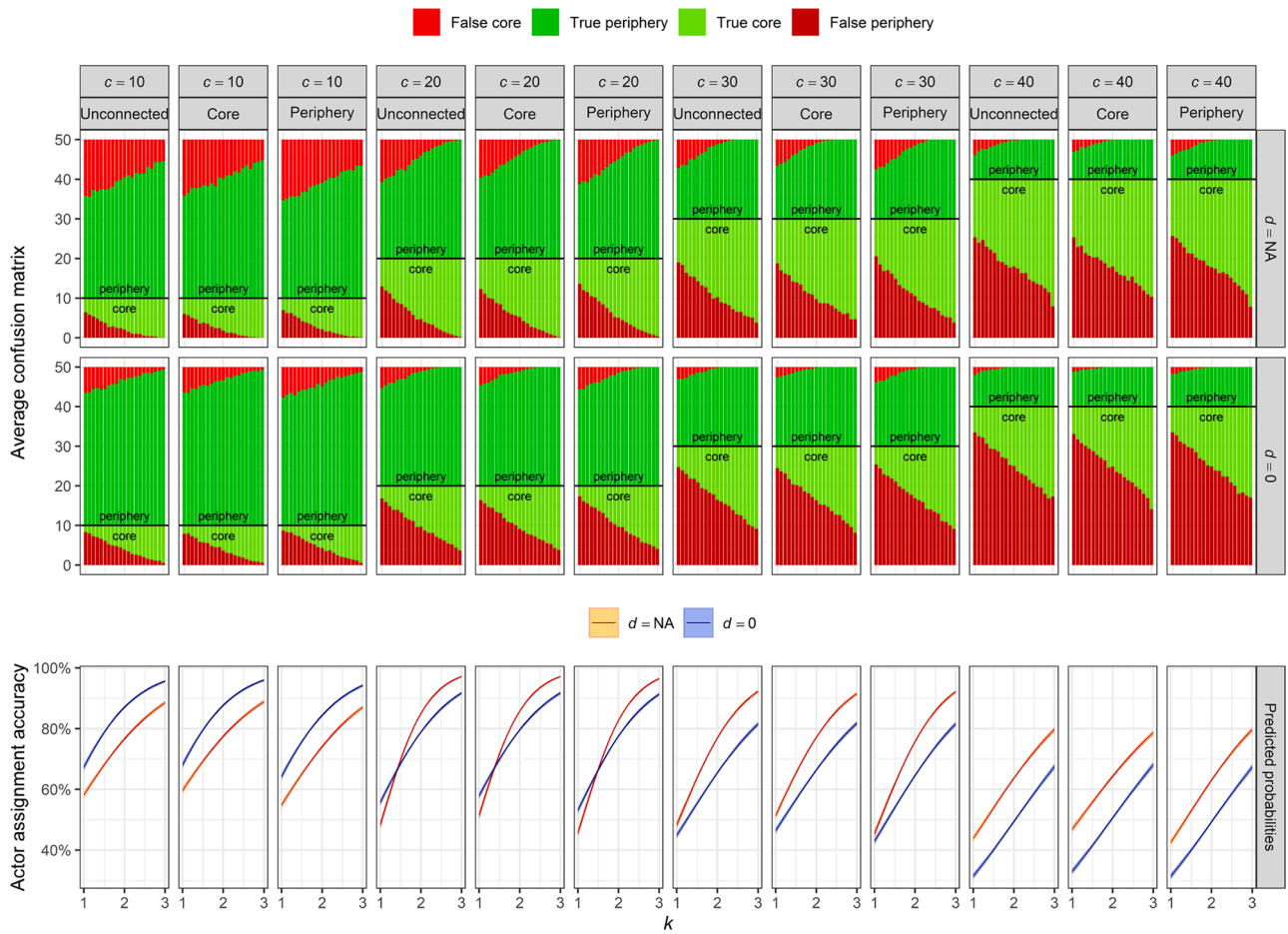


Fig. A4. Performance comparison in core-periphery assignment by core size and approach employed (two-core networks). Top panels show the average confusion matrices for 50 synthetic networks sharing the same k and c , and connection type (unconnected, core-connected, periphery-connected) parameters, employing the standard “setting-to-missing” approach. Middle panels show the average confusion matrices for the same networks using the minimum-density blocks approach with $d = 0$. Bottom panels illustrate the predicted probabilities of correctly classifying all 50 actors within each network. The orange line represents the results obtained using the “setting-to-missing” approach, while the blue line indicates the probabilities from the minimum-density blocks approach with $d = 0$.

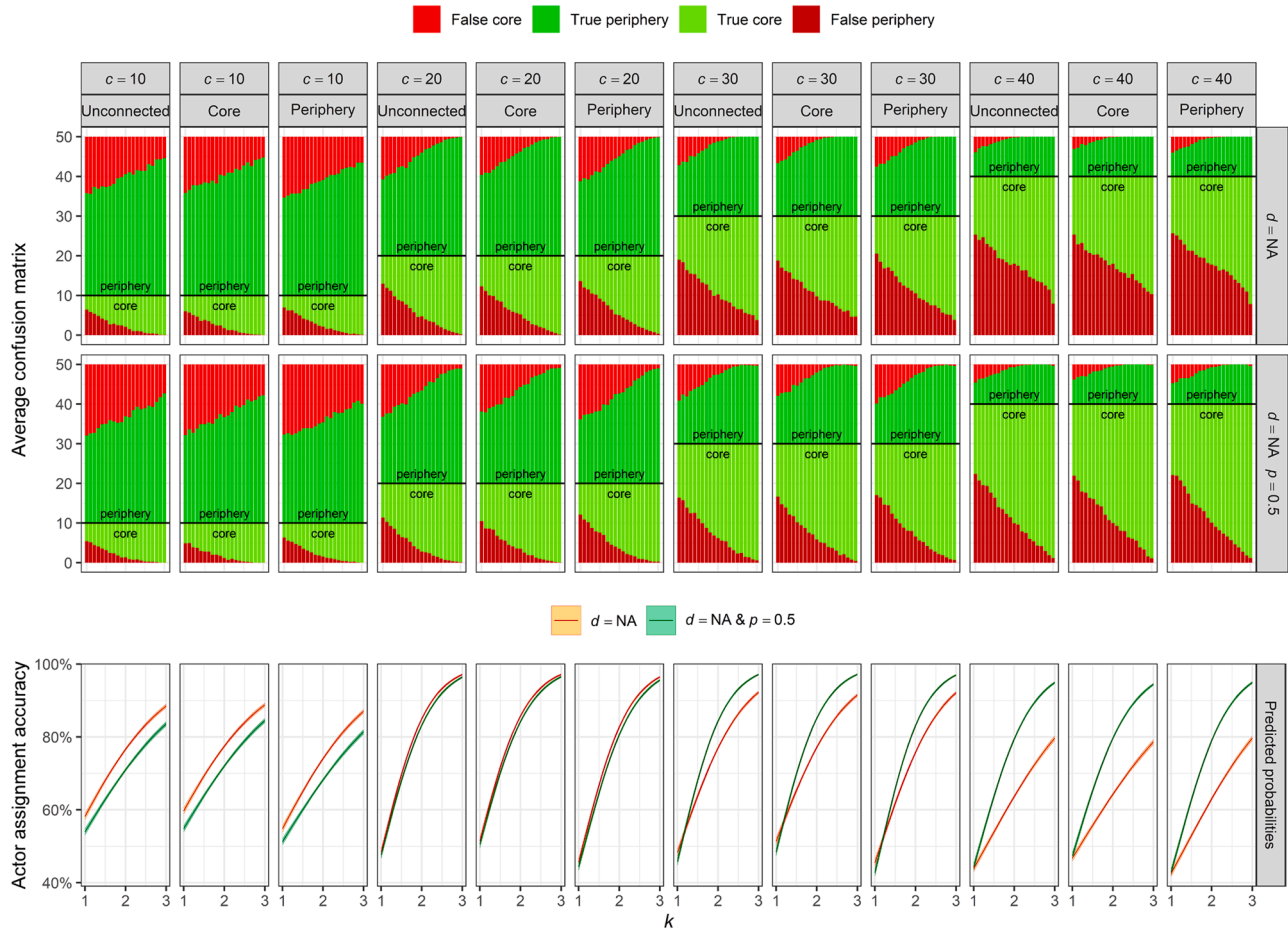


Fig. A5. Performance comparison in core-periphery assignment by core size and approach employed (two-core networks). Top panels show the average confusion matrices for 50 synthetic networks sharing the same k and c , and connection type (unconnected, core-connected, periphery-connected) parameters, employing the standard "setting-to-missing" approach. Middle panels show the average confusion matrices for the same networks using the p -core extension with $p = 0.5$. Bottom panels illustrate the predicted probabilities of correctly classifying all 50 actors within each network. The orange line represents the results obtained using the "setting-to-missing" approach, while the green line indicates the probabilities using the p -core extension with $p = 0.5$.

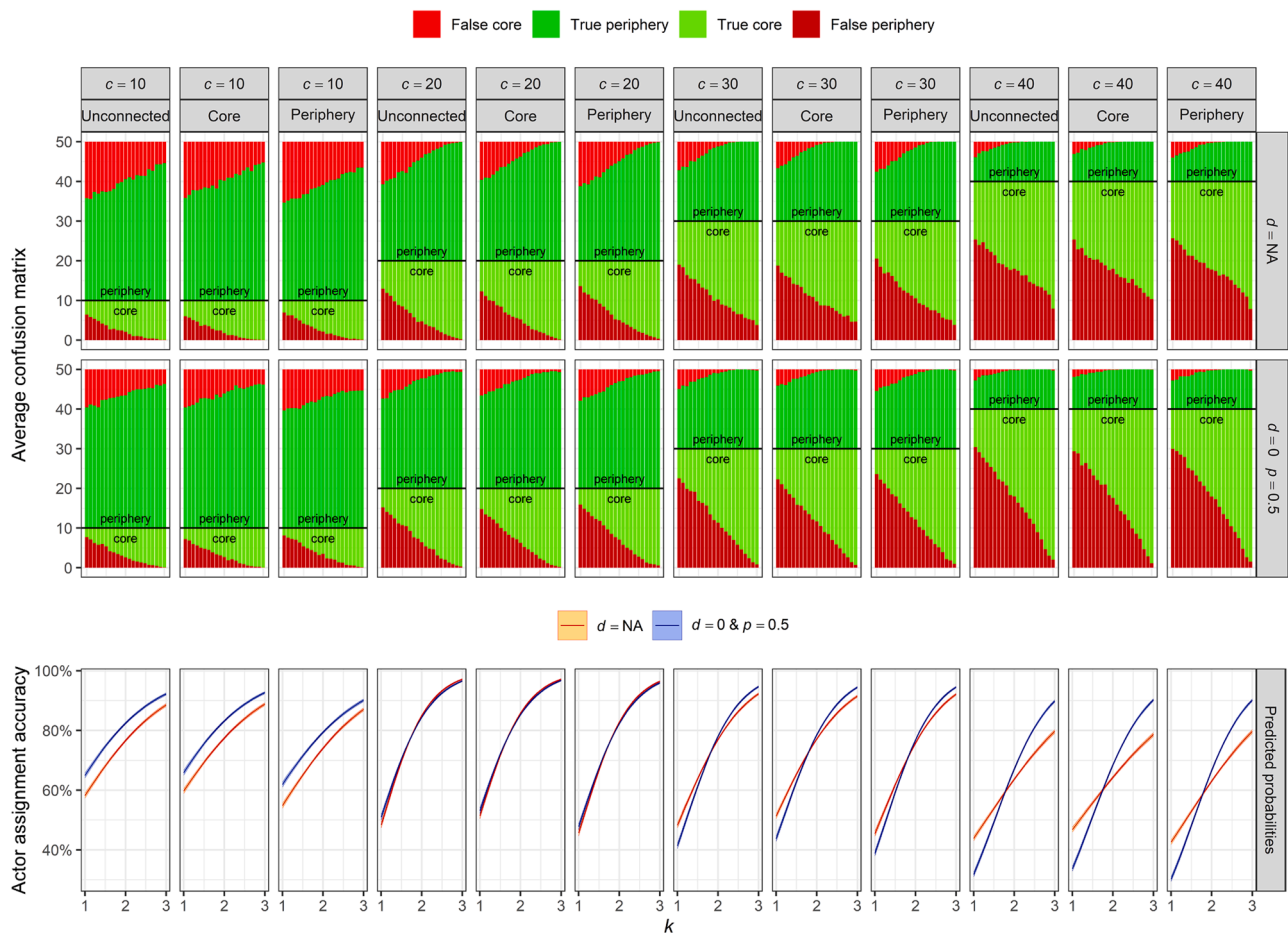


Fig. A6. Performance comparison in core-periphery assignment by core size and approach employed (two-core networks). Top panels show the average confusion matrices for 50 synthetic networks sharing the same k and c , and connection type (unconnected, core-connected, periphery-connected) parameters, employing the standard “setting-to-missing” approach. Middle panels show the average confusion matrices for the same networks using the minimum-density blocks approach with $d = 0$ in combination with the p -core extension with $p = 0.5$. Bottom panels illustrate the predicted probabilities of correctly classifying all 50 actors within each network. The orange line represents the results obtained using the “setting-to-missing” approach, while the blue line indicates the probabilities using the minimum-density blocks approach with $d = 0$ and the p -core extension with $p = 0.5$.

Data availability

The study uses publicly available data sourced from previous publications, as indicated in the text. The implementations have been incorporated into Socnet.se, a cross-platform open-source software client designed for blockmodeling, available here: <https://socnet.se/>, and the Socnet.se commands for reproducing the results are available on the Socnet.se website. All example networks used in this study are also available on this website. All the analyses and figures presented in this article can be recreated using the provided R code, accessible at: https://github.com/joseluisesna/Borgatti-Everett_core-periphery_model_revision.

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